

2. b: 15, 16, 19, ~~11~~ 221

15.

$$L\tilde{c} = \tilde{b}: \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 11 \end{bmatrix}$$

$$c_1 = 2, c_2 = 11 - 4c_1 = 3$$

$$\tilde{c} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$U\tilde{x} = \tilde{c}: \begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$x_2 = 3, x_1 = \frac{1}{2}(2 - 4(3)) = -5$$

$$\tilde{x} = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$$

$$A = LU = \begin{bmatrix} 2 & 4 \\ 8 & 17 \end{bmatrix}$$

$$\left[ \begin{array}{cc|c} 2 & 4 & 2 \\ 8 & 17 & 11 \end{array} \right] \xrightarrow{-4r_1+r_2} \left[ \begin{array}{cc|c} 2 & 4 & 2 \\ 0 & 1 & 3 \end{array} \right]$$

Backsub yields  $\tilde{x} = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$ .

$$16. \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

$$c_1 = 4, c_2 = 5 - 4 = 1, c_3 = 6 - 4 - 1 = 1$$

$$\tilde{c} = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$$

$$x_3 = 1, x_2 = 1 - 1 = 0, x_1 = 4 - 0 - 1 = 3$$

$$\tilde{x} = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

$$19. \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow{-1 \text{ row } 1 + \text{row } 2} \begin{matrix} l_{21} = 1 \\ l_{31} = 0 \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow{-1 \text{ row } 2 + \text{row } 3} l_{32} = 1$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S_0 \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a & a & 0 \\ a & a+b & b \\ 0 & b & b+c \end{bmatrix} \xrightarrow{-r_1+r_2} \begin{matrix} l_{21} = 1 \\ l_{31} = 0 \end{matrix}$$

$$\begin{bmatrix} a & a & 0 \\ 0 & b & b \\ 0 & b & b+c \end{bmatrix} \xrightarrow{-r_2+r_3} l_{32} = 1$$

$$\begin{bmatrix} a & a & 0 \\ 0 & b & b \\ 0 & 0 & c \end{bmatrix} \quad S_0$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \quad (\text{same as above})$$

$$U = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

~~20. The pivots are 2 & 7 since the first ~~two~~ elimination step for A is the same as the elimination step for B.~~