2.5: 1, 2, 3, 6

221

1. Check these.

$$A^{-1} = \begin{bmatrix} 0 & 1/4 \\ 1/3 & 0 \end{bmatrix}$$

$$B^{-1} = \frac{1}{4}\begin{bmatrix} 2 & 0 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ -1 & 1/2 \end{bmatrix}$$

$$C^{-1} = \frac{1}{3\cdot 7 - 4\cdot 5}\begin{bmatrix} 7 & -4 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 7 & -4 \\ -5 & 3 \end{bmatrix}$$

---

2. $$P^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$ check it

$$P^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$ check it

---

3. Extra

$$\begin{bmatrix} 10 & 20 \\ 20 & 50 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 10 & 20 & 1 \\ 20 & 50 & 0 \end{array}\right] \xrightarrow{-2\,\text{row}\,1 + \text{row}\,2}$$

$$\left[\begin{array}{cc|c} 10 & 20 & 1 \\ 0 & 10 & -2 \end{array}\right]$$

$$10x + 20y = 1$$
$$10y = -2$$
$$10x = 1 + 4 \Rightarrow$$

$$\boxed{y = -1/5}$$
$$\boxed{x = 1/2}$$

$$\begin{bmatrix} 10 & 20 \\ 20 & 50 \end{bmatrix}\begin{bmatrix} t \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 10 & 20 & 0 \\ 20 & 50 & 1 \end{array}\right] \xrightarrow{-2r1+r2} \left[\begin{array}{cc|c} 10 & 20 & 0 \\ 0 & 10 & 1 \end{array}\right]$$

$$10t + 20z = 0$$
$$10z = 1 \implies z = 1/10$$
$$t = \frac{1}{10}(-20z) = -\frac{2}{10} = -\frac{1}{5}$$

So

$$A^{-1} = \begin{bmatrix} 1/2 & -1/5 \\ -1/5 & 1/10 \end{bmatrix}$$

Check with formula

$$\begin{bmatrix} 10 & 20 \\ 20 & 50 \end{bmatrix}^{-1} = \frac{1}{500-400}\begin{bmatrix} 50 & -20 \\ -20 & 10 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{5} \\ -\frac{1}{5} & 1/10 \end{bmatrix} \checkmark$$

6(a) Extra $AB = AC$ implies $A^{-1}AB = A^{-1}AC$. Since $A^{-1}A = I$ we have $B = C$.

(b) $B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$

$AB = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $AC = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$,

but $B \neq C$.

2.5 : 22, 23, 28

221

22.

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 2 & 7 & 0 & 1 \end{array}\right] \xrightarrow{-2r1+r2} \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{array}\right] \xrightarrow{-3r2+r1} \left[\begin{array}{cc|cc} 1 & 0 & 7 & -3 \\ 0 & 1 & -2 & 1 \end{array}\right]$$

check:

$$\begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}\begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

23.

$$\left[\begin{array}{ccc|ccc} \multicolumn{3}{c|}{\overbrace{\qquad\qquad}^{A}} & & & \\ 2 & 1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-\frac{1}{2}r_1+r_2}$$

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 0 & 1 & 0 & 0 \\ 0 & \frac{3}{2} & 1 & -\frac{1}{2} & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-\frac{2}{3}r_2+r_3}$$

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 0 & 1 & 0 & 0 \\ 0 & \frac{3}{2} & 1 & -\frac{1}{2} & 1 & 0 \\ 0 & 0 & \frac{4}{3} & \frac{1}{3} & -\frac{2}{3} & 1 \end{array}\right] \xrightarrow{-\frac{3}{4}r_3+r_2}$$

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 0 & 1 & 0 & 0 \\ 0 & \frac{3}{2} & 0 & -\frac{3}{4} & \frac{3}{2} & -\frac{3}{4} \\ 0 & 0 & 4/3 & \frac{1}{3} & -\frac{2}{3} & 1 \end{array}\right] \xrightarrow{-\frac{2}{3}r_2+r_1}$$

$$\left[\begin{array}{ccc|ccc} 2 & 0 & 0 & 3/2 & -1 & 1/2 \\ 0 & \frac{3}{2} & 0 & -3/4 & 3/2 & -3/4 \\ 0 & 0 & 4/3 & 1/3 & -2/3 & 1 \end{array}\right] \xrightarrow[\substack{\frac{2}{3}r_2 \\ 3/4\, r_3}]{\frac{1}{2}r_1}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3/4 & -1/2 & 1/4 \\ 0 & 1 & 0 & -1/2 & 1 & -1/2 \\ 0 & 0 & 1 & 1/4 & -1/2 & 3/4 \end{array}\right]$$

So $A^{-1} = \begin{bmatrix} 3/4 & -1/2 & 1/4 \\ -1/2 & 1 & -1/2 \\ 1/4 & -1/2 & 3/4 \end{bmatrix}$.

Note the similarity w/ A^{-1} from the book/and class example.

28.

$$\left[\begin{array}{cc|cc} 0 & 2 & 1 & 0 \\ 2 & 2 & 0 & 1 \end{array}\right] \xrightarrow{r_1 \leftrightarrow r_2} \left[\begin{array}{cc|cc} 2 & 2 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{array}\right] \xrightarrow{-r_2+r_1} \left[\begin{array}{cc|cc} 2 & 0 & -1 & 1 \\ 0 & 2 & 1 & 0 \end{array}\right] \xrightarrow[1/2\, r_2]{\frac{1}{2}r_1} \left[\begin{array}{cc|cc} 1 & 0 & -1/2 & 1/2 \\ 0 & 1 & 1/2 & 0 \end{array}\right]$$