

Math 221-Exam 1**NAME:** _____

1. Consider the following vectors:

$$\mathbf{u} = \begin{bmatrix} -1 \\ 2 \\ -4 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} -3 \\ -1 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 0 & 2 \\ 1 & 0 & -2 \end{bmatrix},$$

(a) (5 pts) Circle the multiplications that are NOT possible:

$$A\mathbf{u}, \quad \mathbf{u}^T A, \quad B^T \mathbf{v}, \quad BA^T, \quad \mathbf{v}^T A^T.$$

(b) (5 pts) Compute $\mathbf{u} - A\mathbf{v}$.(c) (5 pts) Compute $\|A^T \mathbf{u}\|$.(d) (5 pts) Compute $\mathbf{v} \cdot (B\mathbf{u})$ (note that this is the same as $\mathbf{v}^T B\mathbf{u}$).(e) (5 pts) Compute $A^T A$ and verify that it is symmetric.

(f) (5 pts) Find a vector \mathbf{w} that is perpendicular to \mathbf{u} and then verify that $\mathbf{w} \cdot \mathbf{u} = 0$.

(g) (5 pts) Which of the following type of subspace of \mathbb{R}^3 is the column space of A : $\mathbf{Z} = \{\mathbf{0}\}$, a line through $\mathbf{0}$, a plane through $\mathbf{0}$, or \mathbb{R}^3 . Justify your answer.

(h) (5 pts) Which of the following type of subspace of \mathbb{R}^2 is the column space of B : $\mathbf{Z} = \{\mathbf{0}\}$, a line through $\mathbf{0}$, or \mathbb{R}^2 . Justify your answer.

2. (10 pts) Use elimination and back-substitution to solve the system

$$\begin{aligned}2x + 6y &= 4 \\x + 4y &= 6\end{aligned}$$

3. Let

$$A = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 4 & 4 \\ 0 & 4 & 8 \end{bmatrix}$$

(a) Compute the LU (15 pts) and the LDU (5 pts) factorization of A .

(b) (10 pts) Use the LU factorization of A to solve $A\mathbf{x} = \mathbf{b}$, where $\mathbf{b} = (1, 0, 0)$. Note that for this you first solve $L\mathbf{c} = \mathbf{b}$ and then $U\mathbf{x} = \mathbf{c}$.

4. (a) (15 pts) Compute A^{-1} using Gauss-Jordan elimination, where

$$A = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 3 & 3 \\ 2 & 3 & 4 \end{bmatrix}.$$

- (b) (5 pts) Use A^{-1} to solve $A\mathbf{x} = \mathbf{b}$, where $\mathbf{b} = (1, 0, 0)$.

Extra Credit: Find the values a and b that satisfy the following equation:

$$\begin{bmatrix} 4 & -1 & -1 & -1 \\ -1 & 4 & -1 & -1 \\ -1 & -1 & 4 & -1 \\ -1 & -1 & -1 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} a & b & b & b \\ b & a & b & b \\ b & b & a & b \\ b & b & b & a \end{bmatrix}$$

Hint: Don't use elimination.