§ 11.2

Math 172-Quiz 9 NAME: Key

Instructions: *Show at least one step of your work (where appropriate) for full credit.*

1. Use the ratio test to determine whether the following series is convergent:

$$\sum_{n=1}^{\infty} (-1)^n \frac{2^n}{n!}.$$

Recall that $n! = n \cdot (n-1) \cdot (n-2) \cdots 2 \cdot 1$.

$$\lim_{n\to\infty} \left| \frac{\frac{(-1)^{n+1} 2^{n+1}}{(n+1)!}}{\frac{(-1)^n 2^n}{n!}} \right| = \lim_{n\to\infty} \left| \frac{n!}{(n+1)!} \cdot \frac{2^{n+1}}{2^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{\cancel{n!}}{(n+1)\cancel{n!}} \cdot 2 \right| = \lim_{n\to\infty} \frac{2}{n+1} = 0.$$

Thus the series is absolutely convergent

2. Find the radius of convergence and interval of convergence for the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n^3}.$$

can ignore because of absolute value

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+1-1} x^{n+1}}{(n+1)^3} \cdot \frac{n^3}{(-1)^{n-1} x^n} \right|$$

$$= \lim_{n\to\infty} |x| \cdot \left| \frac{n^3}{(n+1)^3} \right| = |x|$$

$\boxed{R = 1}$

$\Rightarrow$ convergence if $-1 < x < 1$.

$\underline{x = -1}$: $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(-1)^n}{n^3} = \sum_{n=1}^{\infty} \frac{-(-1)^{2n}}{n^3} = -\sum_{n=1}^{\infty} \frac{-1}{n^3}$ p-series convergent

$\underline{x = 1}$: $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3}$ is convergent (alternating p-series).