Math 182-Quiz 10 **NAME:** Key

Instructions: *Show at least one step of your work (where appropriate) for full credit.*

1. Use the series

$$\frac{1}{1-x} = \sum_{n=1}^{\infty} x^n, \quad -1 < x < 1$$

to find the series for $f(x) = \frac{1}{5+x}$.

$$\frac{1}{5+x} = \frac{1}{5} \cdot \frac{1}{1-(-\frac{x}{5})}$$

$$= \frac{1}{5} \sum_{n=0}^{\infty} \left(-\frac{x}{5}\right)^n$$

$$= \sum_{n=0}^{\infty} \frac{1}{5} \cdot (-1)^n \frac{x^n}{5^n}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{5^{n+1}} x^n$$

2. Compute the Taylor series,

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n,$$

for $f(x) = \frac{1}{x}$ centered at $x = 1$.

$f(x) = \frac{1}{x}$ $\qquad$ $f(1) = 1$

$f'(x) = -x^{-2}$ $\qquad$ $f'(1) = -1$

$f''(x) = 2x^{-3}$ $\qquad$ $f''(1) = 2 = 2!$

$f'''(x) = -2 \cdot 3\, x^{-4}$ $\qquad$ $f'''(1) = -2 \cdot 3 = -3!$

$\vdots$

$$\frac{1}{x} = f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \frac{f'''(1)}{3!}(x-1)^3 + \cdots$$

$$= 1 - (x-1) + \frac{\cancel{2!}}{\cancel{2!}}(x-1)^2 + \frac{-\cancel{3!}}{\cancel{3!}}(x-1)^3 + \cdots$$

$$= 1 - (x-1) + (x-1)^2 - (x-1)^3 + \cdots$$