

MATH 172 – Sample Final Exam

1. (40 Points) Show work that evaluates the following integrals. In the case of definite integrals, give the value; in the case of indefinite integrals, give the most general antiderivative. If an integral is improper, compute its value if it converges, or show why it diverges. If you make a u -substitution or perform integration by parts, you must show intermediate work; *e.g.*, *what is u and/or what is dv ?*

(a) (8 Points) $\int x^2 \sin x \, dx$

(b) (8 Points) $\int_1^{e^2} \frac{\ln \sqrt{x}}{\sqrt{x}} \, dx$

(c) (8 Points) $\int_2^{\infty} x e^{-x^2} \, dx$

(d) (8 Points) $\int \frac{1}{\sqrt{9-x^2}} \, dx$

(e) (8 Points) $\int_1^2 \frac{1}{x \ln x} \, dx$ (Hint: When is $\ln x = 0$?)

2. (10-15 Points) A problem from 10.1.

3. (30 Points) Consider the region bounded by the curves $y = e^{-x}$, $x = 0$, $x = 3$, and $y = 0$.

(a) (5pts) Sketch the region.

(b) (10pts) Find the area of the region.

(c) Set up, but do not evaluate, an integral representing the volume of the solid obtained by revolving this region about each of the following lines:

i. (7pts) The y -axis.

ii. (8pts) The line $y = -1$.

4. (20 Points) Consider $f(x) = \cos x$.

(a) (8 Points) Find $T_2(x)$, the second-degree Taylor polynomial for f centered at $x = \pi/2$.

(b) (5 Points) Use $T_2(x)$ to approximate $\cos\left(\frac{87\pi}{180}\right)$.

(c) (7 Points) Give an upper bound on the error made when using $T_2(x)$ to approximate $\cos\left(\frac{87\pi}{180}\right)$.

5. (20 Points) Find the radius of convergence and the interval of convergence of the power

series $\sum_{n=1}^{\infty} \frac{(x+6)^n}{2^n n(n+1)}$.

6. (20 Points) Consider the polar curve

$$r = 2(1 - \sin \theta), \quad 0 \leq \theta \leq 2\pi.$$

- (a) (10pts) Carefully sketch the polar graph of this curve and label the points corresponding to $\theta = 0, \pi/2, \pi, 3\pi/2$.
- (b) (10pts) Find the points at which the tangent to the curve is vertical.
7. (20 Points) Is the series divergent, conditionally convergent, or absolutely convergent? Justify your answer.
- (a) (10 Points) $\sum_{n=1}^{\infty} (-1)^n \frac{2n}{5 + 7n}$
- (b) (10 Points) $\sum_{n=1}^{\infty} \frac{(-4)^n}{n!}$
8. (a) (20-25 Points) Some applied problem(s) involving integration, e.g. center of mass, arc-length, surface area, probability.
9. (15 Points)
- (a) (7pts) Express $f(x) = e^{-x^3}$ as a power series centered at 0. (You may use the techniques of Taylor series or a known series.)
- (b) (8pts) Use the above result to express $\int_0^1 e^{-x^3} dx$ as an infinite series.