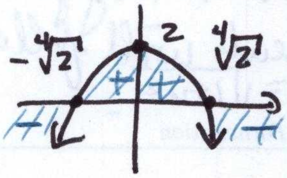
HW 8.8: 12, 16, 18, 20, 57

12. $\int_{-\infty}^{\infty} (2-v^4)\,dv$



Note that

$\int_{-\infty}^{\sqrt[4]{2}} (2-v^4)\,dv$

$= 2v - \frac{v^5}{5}\Big]_{-\infty}^{\sqrt[4]{2}}$

$= \lim_{t\to-\infty} \left(2v - \frac{v^5}{5}\Big]_{t}^{\sqrt[4]{2}} \right)$

$= \lim_{t\to-\infty} \left(2\sqrt[4]{2} - (2^{5/4})/5 - 2t + t^5/5 \right)$

$= -\infty.$

Thus the integral is divergent.

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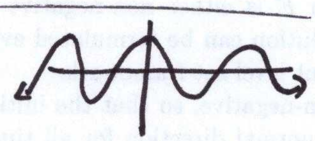
16. $\int_{-\infty}^{\infty} \cos \pi t\,dt$



$\int_0^{\infty} \cos\pi t\,dt = \frac{1}{\pi}\sin\pi t\Big]_0^{\infty}$

$= \lim_{t\to\infty} \frac{1}{\pi}\sin\pi t\Big]$

$= \frac{1}{\pi}\lim_{t\to\infty} \sin\pi t$

NON-CONVEGENT.

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18. $\int_0^{\infty} \frac{dz}{z^2+3z+2}$

$\frac{1}{z^2+3z+2} = \frac{1}{(z+1)(z+2)} = \frac{A}{z+1} + \frac{B}{z+2}$

$\Rightarrow 1 = A(z+2) + B(z+1)$

$= (A+B)z + (2A+B)$

$A+B=0, \quad 2A+B=1$

$A=-B \Rightarrow -2B+B=1$

$\boxed{A=1} \Rightarrow \boxed{B=-1}$

$\int_0^{\infty} \frac{dz}{z^2+3z+2}$

$= \int_0^{\infty} \frac{1}{z+1}dz - \int_0^{\infty}\frac{1}{z+2}dz$

$= \lim_{t\to\infty} \ln|z+1|\Big]_0^t - \lim_{t\to\infty}\ln|z+2|\Big]_0^t$

$= \lim_{t\to\infty} \ln|t+1| - \lim_{t\to\infty}\ln|t+2|$

$+ \ln|2|$

DIVERGENT

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20. $\int_{-\infty}^{6} r e^{r/3}\,dr \qquad \begin{matrix} u=r & dv=e^{r/3}dr \\ du=dr & v=3e^{r/3}\end{matrix}$

$= 3re^{r/3}\Big]_{-\infty}^{6} - \int_{-\infty}^{6} 3e^{r/3}dr$

$= \lim_{t\to-\infty} \left(18e^2 - 3te^{t/3}\right) - 9e^{r/3}\Big]_{-\infty}^{6}$

$= 18e^2 - 3\lim_{t\to-\infty} te^{t/3} - 9e^2 + 9\lim_{t\to-\infty} e^{t/3}$

$= 9e^2$

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57. $\int_0^1 x^{-p}\,dx \qquad p>0$

$= \lim_{t\to0^+} \frac{x^{-p+1}}{(-p+1)}\Big]_t^1$

$= \lim_{t\to0^+} \left(\frac{1}{1-p} - \frac{t^{1-p}}{1-p} \right)$

$= \begin{cases} \frac{1}{1-p} & 1-p>0 \\ \text{Divergent} & 1-p<0 \end{cases}$