HW 8.8: 2, 6, 8, 10, 14, 28, 34

2. (a) ~~proper~~: no infinite limits or discontinuities in [1, 2].

(b) ~~improper~~: $x = \frac{1}{2}$ in [0, 1]

(c) ~~improper~~: infinite limits

(d) ~~improper~~: infinite discontinuity at $x = 1$

6. Divergent:

$$\int_{-\infty}^{0} \frac{1}{2x-5}\,dx \qquad u = 2x-5,\ du = 2dx$$

$$= \int_{-\infty}^{-5} \frac{1}{u}\cdot\frac{1}{2}du$$

$$= \frac{1}{2}\lim_{t\to-\infty}\left[\ln|2x-5|\right]_t^{-5}$$

$$= \frac{1}{2}\lim_{t\to-\infty}\left[\ln 5 - \ln|2t-5|\right]$$

$$= \frac{\ln 5}{2} - \boxed{\frac{1}{2}\lim_{t\to-\infty}\ln|2t-5|} = -\infty.$$

8. $\int_0^\infty \frac{x}{(x^2+2)^2}dx$ Convergent

$u = x^2+2,\ du = 2x\,dx$

$$= \int_2^\infty \frac{1}{2}\cdot\frac{1}{u^2}du = \frac{1}{2}(-u^{-1})\Big]_2^\infty$$

$$= -\frac{1}{2}\lim_{t\to\infty}\left(\frac{1}{u}\Big]_2^t\right)$$

$$= -\frac{1}{2}\left[\frac{1}{t} - \frac{1}{2}\right] = \frac{1}{4}.$$

10. divergent:

$$\int_{-\infty}^{-1} e^{-2t}\,dt$$

$$= -\frac{1}{2}e^{-2t}\Big]_{-\infty}^{-1}$$

$$= \lim_{x\to-\infty} -\frac{1}{2}e^{-2t}\Big]_x^{-1}$$

$$= \lim_{t\to-\infty}\left[-\frac{1}{2}e^{2} + \frac{1}{2}e^{-2x}\right]$$

$$= \infty$$

14. convergent

$$\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx \qquad u = \sqrt{x},\ du = \frac{1}{2\sqrt{x}}dx$$

$$= 2\int_1^\infty e^{-u}du$$

$$= 2\lim_{t\to\infty} -e^{-u}\Big]_1^t$$

$$= 2\lim_{t\to\infty}\left[-e^{-t} + e^{-1}\right]$$

$$= 2e^{-1}$$

34. $$\int_0^1 \frac{1}{4y-1}dy = \int_0^{1/4}\frac{1}{4y-1}dy + \int_{1/4}^1 \frac{1}{4y-1}dy$$

$$= \lim_{t\to 1/4^-}\left[\frac{1}{4}\ln|4y-1|\Big]_0^{1/4}\right] + \lim_{t\to 1/4^+}\left[\frac{1}{4}\ln|4y-1|\Big]_{1/4}^{1}\right]$$

$$= \frac{1}{4}\lim_{t\to 1/4^-}\ln|4y-1| - \frac{1}{4}\lim_{t\to \frac{1}{4}^+}\ln|4y-1| + \frac{1}{4}\ln 3.$$

$$= -\infty + \infty + \frac{1}{4}\ln 3.\ \text{divergent.}$$