HW §8.1: 20, 22, 24, 32

20. $\int_0^1 (x^2+1)e^{-x}\,dx$

$u = x^2+1 \quad dv = e^{-x}dx$

$du = 2x\,dx \quad v = -e^{-x}$

$= (x^2+1)(-e^{-x})\Big|_0^1 - \int (-e^{-x})\,2x\,dx$

$= (-2e^{-1}+1) + 2\int_0^1 x e^{-x}\,dx$ $\quad \boxed{u=x,\ du=dx,\ dv=e^{-x},\ v=-e^{-x}}$

$= (1-2e^{-1}) + 2\left(-xe^{-x}\Big|_0^1 + \int_0^1 e^{-x}dx\right)$

$= \left(1-2e^{-1}-2e^{-1} + \left(-e^{-x}\Big|_0^1\right)\right)$

$= 1-4e^{-1}-e^{-1}+1$

$= \cancel{(2-e^{-1})}\ 2-5e^{-1}.$

22. $\int_4^9 \frac{\ln y}{\sqrt{y}}\,dy = \int_4^9 (\ln y)\,y^{-\frac{1}{2}}\,dy$

$u = \ln y \quad dv = y^{-\frac{1}{2}}dy$

$du = \frac{1}{y}dy \quad v = 2y^{\frac{1}{2}}$

$= \ln y\,(2y^{1/2})\Big|_4^9 - \int_4^9 2y^{1/2}y^{-1}\,dy$

$= (\ln 9)\cdot 6 - (\ln 4)\cdot 4 - 2\int_4^9 y^{-1/2}\,dy$

$= 6\ln 9 - 4\ln 4 - 2\cdot 2y^{1/2}\Big|_4^9$

$= 6\ln 9 - 4\ln 4 - 12 + 8.$

$= 6\ln 9 - 4\ln 4 - 4.$

24. $\int_0^\pi x^3\cos x\,dx$

$u = x^3 \quad dv = \cos x\,dx$

$du = 3x^2dx \quad v = +\sin x$

$x^3\sin x\Big|_0^\pi - \int_0^\pi (\sin x)\,3x^2\,dx$

$= -3\int_0^\pi x^2\sin x\,dx$ $\quad u = x^2 \quad dv = \sin x\,dx$, $\ du = 2x\,dx \quad v = -\cos x$

$= -3\left(x^2(-\cos x)\Big|_0^\pi - \int_0^\pi(-\cos x)\,2x\,dx\right)$

$= -3\left(\pi^2 + 2\int_0^\pi x\cos x\,dx\right)$ $\quad u=x,\ du=dx,\ dv=\cos x\,dx,\ v=\sin x$

$= -3\pi^2 + 2\left(x\sin x\Big|_0^\pi - \int_0^\pi \sin x\,dx\right)$

$= -3\pi^2 + \cos x\Big|_0^\pi$

$= -3\pi^2 - 1 - 1 = -3\pi^2 - 2.$

32. $\int_0^t e^s\sin(t-s)\,ds$

Let's compute the indefinite integral first.

$\int e^s\sin(t-s)\,ds \quad dv = e^s ds \quad u = \sin(t-s)$

$v = e^s \quad du = -\cos(t-s)\,ds$

$= e^s\sin(t-s) + \int e^s\cos(t-s)\,ds$ $\quad \left\{ dv = e^s ds,\ v = e^s,\ u = \cos(t-s),\ du = \sin(t-s)\,ds \right.$

$= e^s\sin(t-s) + e^s\cos(t-s) - \int e^s\sin(t-s)\,ds$

So $\int e^s\sin(t-s)\,ds = \frac{1}{2}e^s\left(\sin(t-s)+\cos(t-s)\right) + C$

Thus $\int_0^t e^s\sin(t-s)\,ds$

$= \frac{1}{2}e^s\left(\sin(t-s)+\cos(t-s)\right)\Big|_0^t$

$= \frac{1}{2}\left(e^t - \sin t - \cos t\right)$