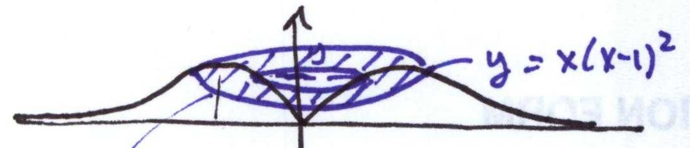
HW 6.3: 1, 2, 3, 6, 10, 37, 38

1.



~~Volume of~~ $A(y)$ = area of slice

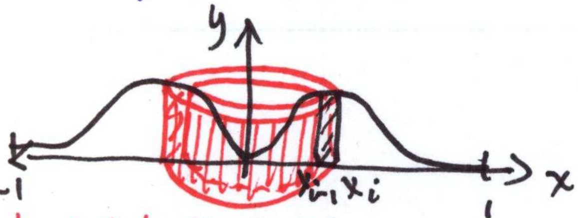
$= \pi(\text{outer radius})^2 - \pi(\text{inner})^2$

To find outer & inner radii, must solve

$y = x(x-1)^2$

for x in terms of y. Yuck!

Instead, use shells.



height $= f(\bar{x}_i)$

circumferance $= 2\pi \bar{x}_i$

So $V = 2\pi \int_0^1 x f(x)\,dx$

$= 2\pi \int_0^1 x^2(x^2-2x+1)\,dx$

$= 2\pi \int_0^1 x^4 - 2x^3 + x^2\,dx$

$= 2\pi\left(\frac{1}{5} - \frac{1}{2} + \frac{1}{3}\right)$.

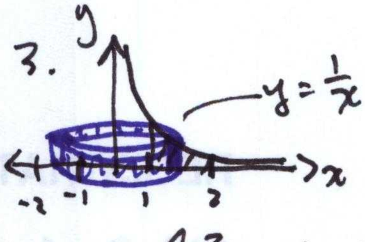
#2 You sketch a shell. Cylindrical shells are preferrable here because $y = \sin(x^2)$ can't be solved for x easily.

$V = 2\pi \int_0^{\sqrt{\pi}} x \sin(x^2)\,dx$

$u = x^2$, $du = 2x\,dx \Rightarrow x\,dx = \frac{1}{2}du$

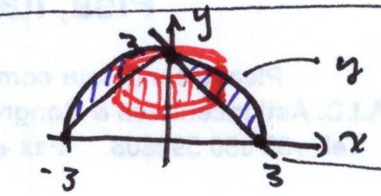
$V = \frac{2\pi}{2}\int_0^{\pi} \sin u\,du = -\pi \cos u \Big]_0^{\pi}$

3.



$V = \int_1^2 x \cdot \frac{1}{x}\,dx = \int_1^2 dx = x\Big]_1^2 = 1.$

6



$y = 3 + 2x - x^2$

$y = -x + 3$

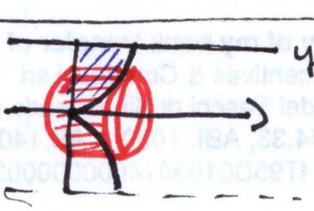
$V = 2\pi \int_0^3 x(\cancel{3} + 2x - x^2 - (-x + \cancel{3}))\,dx$

$= 2\pi \int_0^3 (3x^2 - x^3)\,dx$

$= 2\pi\left(x^3 - \frac{x^4}{4}\right)\Big|_0^3 = 2\pi(3)^3 - \frac{\pi}{2}(3)^4$

$= 54\pi - \frac{81}{2}\pi$

10.



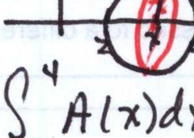
$V = 2\pi \int_0^1 y\sqrt{y}\,dy$

$= 2\pi \int_0^1 y^{3/2}\,dy$

$= 2\pi \cdot \frac{2}{5} \cdot y^{5/2}\Big]_0^1$

$= \frac{4\pi}{5}$.

38. $x^2 - 6x + 8 = 0$, $(x-4)(x-2) = 0$



$V = \int_2^4 A(x)\,dx = \int_2^4 \pi(x^2 - 6x + 8)^2\,dx$

= you compute the integral