: 12.10 6, 8, 14, 18, 30, 32

$f(0) = \ln(1) = 0$

$f'(x) = \frac{1}{1+x} \Rightarrow f'(0) = 1$

$f''(x) = \frac{-1}{(1+x)^2} \Rightarrow f''(0) = -1$

$f'''(x) = \frac{2}{(1+x)^3} \Rightarrow f'''(0) = 2$

$f^{(4)}(x) = -\frac{2\cdot 3}{(1+x)^4} \Rightarrow f^{(4)}(0) = -3\cdot 2$

$\vdots$

$f^{(n)}(x) = (-1)^{n-1}(n-1)!$

So $\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}(n-1)!}{n!} x^n$

$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$

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8. $f(0) = \cos(0) = 1$

$f'(x) = -3\sin 3x \Rightarrow f'(0) = 0$

$f''(x) = -9\cos 3x \Rightarrow f''(0) = -9$

$f'''(x) = 27\sin 3x \Rightarrow f'''(0) = 0$

$f^{(4)}(x) = 81\cos 3x \Rightarrow f^{(4)}(0) = 81$

$\vdots$

$\cos 3x = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$

$\cos(3x) = 1 - \frac{9}{2!}x^2 + \frac{81}{4!}x^4 - \cdots$

$= \sum_{n=0}^{\infty} \frac{3^{2n}}{(2n)!}(-1)^n x^{2n}$

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14. $f(-2) = -2-(-8) = 6$

$f'(x) = 1-3x^2,\ f'(-2) = 1-12 = -11$

$f''(x) = -6x,\ f''(-2) = 12$

$f^{(n)} = 0$ for $n \geq 4$.

So

$f(x) = \frac{f(-2)}{1} + \frac{f'(-2)}{1}(x+2) + \frac{f''(-2)}{2}(x+2)^2$

$+ \frac{f'''(-2)}{6}(x+2)^3 + 0$

$= 6 - 11(x+2) + 12(x+2)^2$

$-6(x+2)^3.$

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18. $f(\frac{\pi}{2}) = \sin\frac{\pi}{2} = 1$

$f'(x) = \cos x,\ f'(\frac{\pi}{2}) = 0$

$f''(x) = -\sin x,\ f''(\frac{\pi}{2}) = -1$

$f'''(x) = -\cos x,\ f'''(\frac{\pi}{2}) = 0$

$f^{(4)}(x) = \sin x,\ f^{(4)}(\frac{\pi}{2}) = 1.$

$\vdots$

$\sin x = 1 - \frac{(x-\pi/2)^2}{2!} + \frac{(x-\pi/2)^4}{4!}$

$= \sum_{n=0}^{\infty} \frac{(x-\pi/2)^{2n}}{(2n)!}(-1)^n$

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30. $\cos(\frac{\pi}{2}x) =$

$\sum_{n=0}^{\infty} (-1)^n \frac{(\frac{\pi}{2}x)^{2n}}{2n!} = \sum_{n=0}^{\infty} (-1)^n \left(\frac{\pi}{2}\right)^{2n} \frac{x^{2n}}{(2n)!}$

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32. $e^x + 2e^{-x} =$

$\sum_{n=0}^{\infty} \frac{x^n}{n!} + 2\sum_{n=0}^{\infty} (-1)^n \frac{x^n}{n!}$

$= \sum_{n=0}^{\infty} (1+2(-1)^n) \frac{x^n}{n!}$