

12.1: 4, 8, 10, 12, 15, 18 - 28 even, 32, 36

4. $\frac{2}{2}, \frac{3}{5}, \frac{4}{8}, \frac{5}{11}, \frac{6}{14}$

~~$\frac{4}{2} = 2, \frac{4}{3} = \frac{4}{3}, \frac{4}{4} = 1, \frac{4}{5} = \frac{4}{5}, \frac{4}{6} = \frac{2}{3}$~~

8. $a_1 = 4, a_2 = \frac{4}{4-1} = \frac{4}{3}$,

$a_2 = \frac{4}{3} / (\frac{4}{3} - 1) = \frac{4}{3} / \frac{1}{3} = 4$.

$a_4 = \frac{4}{4-1} = \frac{4}{3}, a_5 = 4$.

10. $a_n = (\frac{1}{3})^{n-1}$.

12. $a_n = (-1)^{n-1} \cdot \frac{n}{(n+1)^2}$

15. $a_1 = \frac{1}{3}, a_2 = \frac{2}{5}, a_3 = \frac{3}{7}$,

$a_4 = \frac{4}{9}, a_5 = \frac{5}{11}$.

Seems to be approaching $\frac{1}{2}$.

$\lim_{n \rightarrow \infty} \frac{n}{2n+1} \cdot \frac{1/n}{1/n} = \lim_{n \rightarrow \infty} \frac{1}{2+1/n}$

$= \frac{1}{2}$.

18. $a_n = \frac{n^3}{n^3+1}$

$\lim_{n \rightarrow \infty} \frac{n^3}{n^3+1} \cdot \frac{1/n^3}{1/n^3} = \lim_{n \rightarrow \infty} \frac{1}{1+1/n^3} = 1$.

20. $\lim_{n \rightarrow \infty} \frac{n^3}{n+1} \cdot \frac{1/n}{1/n}$

$= \lim_{n \rightarrow \infty} \frac{n^2}{1+1/n} = (\lim_{n \rightarrow \infty} n^2) / 1 = \infty$

22. $\lim_{n \rightarrow \infty} \frac{3^{n+2}}{5^n} = \lim_{n \rightarrow \infty} \left(\frac{3^2 \cdot 3^n}{5^n} \right)$

$= 9 \lim_{n \rightarrow \infty} \left(\frac{3}{5} \right)^n = 0$.

24. $\lim_{n \rightarrow \infty} \sqrt{\frac{n+1}{9n+1}}$

$= \sqrt{\lim_{n \rightarrow \infty} \frac{n+1}{9n+1} \cdot \frac{1/n}{1/n}}$

$= \sqrt{\lim_{n \rightarrow \infty} \frac{1+1/n}{9+1/n}} = \sqrt{\frac{1}{9}} = \frac{1}{3}$

26. $\lim_{n \rightarrow \infty} \frac{(-1)^n n^3}{n^3+2n^2+1} \cdot \frac{1/n^3}{1/n^3}$

$= \lim_{n \rightarrow \infty} \frac{(-1)^n}{1+2/n+1/n^3} = \lim_{n \rightarrow \infty} (-1)^n$

DIVERGENT.

28. $\lim_{n \rightarrow \infty} \cos(2/n) = \cos(\lim_{n \rightarrow \infty} \frac{2}{n})$

$= \cos(0) = 1$

32. $\lim_{n \rightarrow \infty} \frac{\ln n}{\ln 2n} \left(\frac{\infty}{\infty} \right)$

$= \lim_{n \rightarrow \infty} \left(\frac{1}{n} \right) / \left(\frac{2}{2n} \right) = \lim_{n \rightarrow \infty} \frac{n}{n} = 1$.

36. $\lim_{n \rightarrow \infty} \ln(n+1) - \ln(n)$

$= \lim_{n \rightarrow \infty} \ln\left(\frac{n+1}{n}\right) = \ln\left(\lim_{n \rightarrow \infty} \frac{n+1}{n}\right)$

$= \ln(1) = 0$.