Calculus II, Exam 3 Review

1. 12.4, Comparison Tests: let a_n and b_n be positive sequences.
   (a) test 1:
      i. if $a_n \geq b_n$ for all n (eventually), and $\sum b_n = \infty$, then $\sum a_n = \infty$;
      ii. if $a_n \leq b_n$ for all n (eventually), and $\sum b_n$ is convergent, then $\sum a_n$ is convergent.
   (b) test 2: if $\lim_{n\to\infty} \frac{a_n}{b_n} = c$ where $c > 0$ is a finite number, then either both series converge or both diverge.
2. 12.5, Alternating Series Test: consider $\sum_{n=1}^{\infty}(-1)^{n-1}b_n$, where $b_n > 0$, then
   (a) if $b_{n+1} \leq b_n$ and $\lim_{n\to\infty} b_n = 0$, the series is convergent.
   (b) $(-1)^{n-1}$ can be replace by $(-1)^n$, $(-1)^{n+1}$, etc.
   (c) Also, you may use the following result: if $\lim_{n\to\infty} b_n \neq 0$, the alternating series is divergent.
3. 12.6, The Ratio and Root Tests. These involve computing the limits
$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|, \quad \lim_{n\to\infty}\sqrt[n]{|a_n|}.$$
4. 12.8, Power Series: determining convergence using the ratio test, computing the radius of convergence and the interval of convergence.
5. 12.9, Representing Functions as Power Series: computing series representations of functions from the series
$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n,$$
   e.g., $\ln(1-x)$, $\tan^{-1}(x)$, $\frac{1}{10+x}$, $\tan^{-1}(x/3)$, $\ln(1+x^2)$. Also, differentiation and integration of power series.
6. 12.10, Taylor and McLaurin Series. Computing power series for more general functions. Also the remaining "important" series: $\cos(x)$, $\sin(x)$ e^x.
7. 12.11, computing Taylor Series approximations of functions.
8. 11.1, sketching graphs of parametric curves, and graphing parametric curves with your calculator.
9. 11.2, computing slopes (first derivative) for parametric curves, also concavity (second derivative) and arclength.