HW 8.4 : 2a, 3b, 7, 8, 10, 12, 16

2. a) $\frac{x}{x^2+x-2} = \frac{x}{(x+2)(x-1)}$

$= \frac{A_1}{x+2} + \frac{A_2}{x-1}$

3 b) $\frac{1}{(x^2-9)^2} = \frac{1}{(x+3)^2(x-3)^2}$

$= \frac{A_1}{x+3} + \frac{A_2}{(x+3)^2} + \frac{A_3}{(x-3)} + \frac{A_4}{(x-3)^2}.$

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7. $\int \frac{x}{x-6}\,dx$ improper

$\frac{x}{x-6} \Rightarrow x-6 \overset{1}{\overline{)\,x+0}}$ $\quad \frac{-(x-6)}{6}$ $\Rightarrow \frac{x}{x-6} = 1 + \frac{6}{x-6}$

$\int \frac{x}{x-6}\,dx = \int dx + \int \frac{6}{x-6}\,dx$

$= x + 6\ln|x-6| + C.$

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8. $\int \frac{r^2}{r+4}\,dr$ $\quad r+4 \overset{r-4}{\overline{)\,r^2+0r+0}}$

$-(r^2+4r)$

$-4r+0$

$-(-4r+16)$

16

$\frac{r^2}{r+4} = r-4+\frac{16}{r+4}$

$\int \frac{r^2}{r+4}\,dr = \int (r-4) + \int \frac{16}{r+4}\,dr$

$= \frac{r^2}{2} - 4r + 16\ln|r+4| + C.$

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10. $\int \frac{1}{(t+4)(t-1)}\,dt$

$\frac{1}{(t+4)(t-1)} = \frac{A_1}{t+4} + \frac{A_2}{t-1}$

$1 = A_1(t-1) + A_2(t+4) \Rightarrow A_1 = -\frac{1}{5}$

So $\int \frac{1}{(t+4)(t-1)}\,dt = \int -\frac{1}{5}\cdot\frac{1}{t+4}\,dt$

$+ \int \frac{1}{5}\cdot\frac{1}{t-1}\,dt$

$= -\frac{1}{5}\ln|t+4| + \frac{1}{5}\ln|t-1| + C.$

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12. $\int_0^1 \frac{x-1}{x^2+3x+2}\,dx = \int_0^1 \frac{x-1}{(x+1)(x+2)}\,dx$

$\frac{x-1}{(x+1)(x+2)} = \frac{A_1}{x+1} + \frac{A_2}{x+2}$

$\Rightarrow x-1 = A_1(x+2) + A_2(x+1)$

$\Rightarrow -3 = -A_2 \Rightarrow \boxed{A_2 = 3}$

$\Rightarrow -2 = A_1 \Rightarrow \boxed{A_1 = -2}$

$\int_0^1 \frac{x-1}{(x+1)(x+2)}\,dx = -2\int_0^1 \frac{dx}{x+1} + 3\int_0^1 \frac{1}{x+2}\,dx$

$= -2\ln|x+1| + 3\ln|x+2| + C.$

~~16. $x^2-x-6 \overset{x}{\overline{)\,x^3-4x-10}}$ x^2-~~

16. $x^2-x-6 \overset{x+1}{\overline{)\,x^3+0x^2-4x-10}}$

$-(x^3-x^2-6x)$

$x^2+2x-10$

$-(x^2-x-6)$

$3x-4$

$\int_0^1 \frac{x^3-4x-10}{x^2-x-6}\,dx = \int_0^1 x+1\,dx + \int_0^1 \frac{3x-4}{x^2-x-6}\,dx$

$\frac{3x-4}{x^2-x-6} = \frac{3x-4}{(x-3)(x+2)} = \frac{A_1}{x-3} + \frac{A_2}{x+2}$

$3x-4 = A_1(x+2) + A_2(x-3)$

$\underline{x=-2}$: $-10 = -5A_2 \Rightarrow \boxed{A_2 = 2}$

$\underline{x=3}$: $5 = 5A_1 \Rightarrow A_1 = 1$

$\hookrightarrow = \frac{x^2}{2} + x\Big|_0^1 + \int_0^1 \frac{1}{x-3}\,dx + \int_0^1 \frac{2}{x+2}\,dx$

$+ \ln 2 - \ln 3 + 2\ln 3 - 2\ln 2 = \frac{3}{2} - \ln 2 + \ln 3.$