8.3: 2, 6, 8 (also, see the int tables 21-46)

2. $\int x^3\sqrt{9-x^2}\,dx$; $x = 3\sin\theta$

$dx = 3\cos\theta\,d\theta$

$= \int 27\sin^3\theta\sqrt{9-9\sin^2\theta}\,(3\cos\theta\,d\theta)$

$9\cos^2\theta$

$= \int 81\sin^3\theta\cdot 3\cos^2\theta\,d\theta$

$= 243\int \sin^2\theta\cdot\cos^2\theta\,(\sin\theta\,d\theta)$

$= 243\int (1-\cos^2\theta)\cos^2\theta(\sin\theta\,d\theta)$

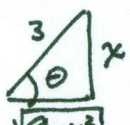
$u = \cos\theta$, $du = -\sin\theta\,d\theta$

$= -243\int (1-u^2)u^2\,du$

$= -243\int u^2 - u^4\,du$

$= -243\left(\frac{u^3}{3} - \frac{u^5}{5}\right) + C$

$= -243\left(\frac{\cos^3\theta}{3} - \frac{\cos^5\theta}{5}\right) + C$



$x = 3\sin\theta$

$\sin\theta = \frac{x}{3}$

$\Rightarrow \cos\theta = \frac{\sqrt{9-x^2}}{3}$

$= -243\left(\frac{1}{81}(9-x^2)^{3/2} - \frac{1}{3^5}\cdot\frac{1}{5}(9-x^2)^{5/2}\right) + C$

$= -3^5\left(\frac{1}{3^4}(9-x^2)^{3/2} - \frac{1}{3^5}\cdot\frac{1}{5}(9-x^2)^{5/2}\right) + C$

$= -3(9-x^2)^{3/2} + \frac{1}{5}(9-x^2)^{5/2} + C.$

6. $\int_1^2 \frac{\sqrt{x^2-1}}{x}\,dx$ $\quad x = \sec\theta$

$dx = \sec\theta\tan\theta\,d\theta$

$= \int_{\sec^{-1}1}^{\sec^{-1}2} \frac{\sqrt{\sec^2\theta - 1}}{\sec\theta}\cdot\sec\theta\tan\theta\,d\theta$

$= \int_{\cos^{-1}1}^{\cos^{-1}(\frac{1}{2})} \tan\theta\cdot\tan\theta\,d\theta$

$= \int_0^{\pi/3} \tan^2\theta\,d\theta$ use table ↙

$= \int_0^{\pi/3} (\sec^2\theta - 1)\,d\theta$

$= \tan\theta - \theta\,\Big|_0^{\pi/3}$

$= \tan\pi/3 - \pi/3 - \tan 0$

$= \frac{\sqrt{3}/2}{1/2} - \pi/3 = \sqrt{3} - \pi/3.$

8. $\int \frac{x^3}{\sqrt{x^2+100}}\,dx$ $\quad x = 10\tan\theta$

$dx = 10\sec^2\theta\,d\theta$

$= \int \frac{10^3\tan\theta\cdot 10\sec^2\theta}{\sqrt{100+100\tan^2\theta}}\,d\theta$

$= \int \frac{10^3\tan^3\theta\cdot\sec^2\theta}{10\sec\theta}\,d\theta$

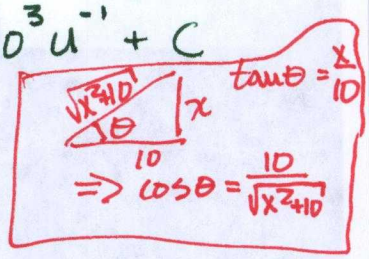
$= 10^3\int \tan\theta\sec\theta\,d\theta$

$= 10^3\int \frac{\sin\theta}{\cos^2\theta}\,d\theta$ $\quad u = \cos\theta$, $du = -\sin\theta\,d\theta$

$= -10^3\int u^{-2}\,du = 10^3 u^{-1} + C$

$= 10^3(\cos\theta)^{-1} + C$

$= 10^3\left(\frac{10}{\sqrt{x^2+10}}\right)^{-1} + C$



$\Rightarrow \cos\theta = \frac{10}{\sqrt{x^2+10}}$