HW: 6, 9, 14 (8.4).

8.3

6. $\displaystyle\int_1^2 \frac{\sqrt{x^2-1}}{x}\,dx$ $x = \sec\theta$, $dx = \sec\theta\tan\theta\,d\theta$

$= \displaystyle\int_{\sec^{-1}(1)}^{\sec^{-1}(2)} \frac{\sqrt{\sec^2\theta-1}}{\cancel{\sec\theta}}\,\cancel{\sec\theta}\tan\theta\,d\theta$

$= \displaystyle\int_{\cos^{-1}(1)}^{\cos^{-1}(\frac{1}{2})} \tan\theta\cdot\tan\theta\,d\theta$

$= \displaystyle\int_0^{\pi/3} \tan^2\theta\,d\theta$

$= \Big\{ \tan\theta - \theta \Big]_0^{\pi/3}$ ← integral table

$= \tan\pi/3 - \pi/3$

$= (\sqrt{3}/2)/(1/2) - \pi/3 = \sqrt{3} - \pi/3.$

---

9. $\displaystyle\int \frac{dx}{\sqrt{x^2+16}}$ $x = 4\tan\theta$, $dx = 4\sec^2\theta\,d\theta$

$= \displaystyle\int \frac{4\sec^2\theta}{\sqrt{16\tan^2\theta+16}}\,d\theta$

$= \displaystyle\int \frac{4\sec^2\theta}{4\sec\theta}\,d\theta$

$= \displaystyle\int \sec\theta\,d\theta$

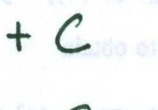
~~$= \tan\theta + C$~~

~~$= \frac{x}{4} + C$~~

$= \ln|\sec\theta + \tan\theta| + C$

$= \ln\left|\frac{\sqrt{x^2+16}}{4} + \frac{x}{4}\right| + C.$

$\tan\theta = \frac{x}{4}$



$\sec\theta = \frac{1}{\cos\theta} = \sqrt{x^2+16}/4$

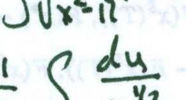
14. $\displaystyle\int \frac{du}{u\sqrt{5-u^2}}$ $u = \sqrt{5}\sin\theta$, $du = \sqrt{5}\cos\theta\,d\theta$

$= \displaystyle\int \frac{\cancel{\sqrt{5}}\cos\theta\,d\theta}{\cancel{\sqrt{5}}\sin\theta\sqrt{5-5\sin^2\theta}}$

$= \displaystyle\int \frac{\cos\theta\,d\theta}{\sin\theta\sqrt{5}\cos\theta}$

$= \frac{1}{\sqrt{5}}\displaystyle\int \csc\theta\,d\theta$

$= \frac{1}{\sqrt{5}}\ln|\csc\theta - \cot\theta| + C$



$\sin\theta = \frac{u}{\sqrt{5}}$

$\cot\theta = \frac{\sqrt{5-u^2}}{u}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{\sqrt{5}}{u}$

$= \frac{1}{\sqrt{5}}\ln\left|\frac{\sqrt{5}}{u} - \frac{\sqrt{5-u^2}}{u}\right| + C.$

~~17. $\displaystyle\int \frac{x}{\sqrt{x^2-7}}\,dx$ $x = (\sec\theta)\sqrt{7}$, $dx = \sqrt{7}\sec\theta\tan\theta\,d\theta$~~

~~$= \displaystyle\int \frac{\sqrt{7}\sec\theta}{\sqrt{7}\tan\theta}\,\sqrt{7}\sec\theta\tan\theta\,d\theta$~~

~~$= \displaystyle\int \sqrt{7}\sec^2\theta\,d\theta$~~

I've made it too hard

---

17. $\displaystyle\int \frac{x}{\sqrt{x^2-17}}\,dx$ $u = x^2-17$, $du = 2x\,dx$

$= \frac{1}{2}\displaystyle\int \frac{du}{u^{1/2}} = u^{1/2} + C$

$= (x^2-17)^{1/2} + C.$

---

20. Same trick as for #17.