

12.9: 4, 6, 14, 18, 26

$$4. f(x) = \frac{3}{1-(x^2)^2} \quad |x^2| < 1$$

$$= 3 \cdot \sum_{n=0}^{\infty} (x^2)^n \quad -1 < x < 1$$

$$= 3 \sum_{n=0}^{\infty} x^{2n} \quad -1 < x < 1$$

$$= 3(1 + x^2 + x^4 + \dots)$$

$$= 3 + 3x^2 + 3x^4 + \dots \quad \boxed{-1 < x < 1}$$

$$6. f(x) = \frac{1}{x+10} = \frac{1}{10(1+\frac{x}{10})}$$

$$= \frac{1}{10} \cdot \frac{1}{1-(-\frac{x}{10})}$$

$$= \frac{1}{10} \sum_{n=0}^{\infty} (-\frac{x}{10})^n \quad |\frac{x}{10}| < 1$$

$$= \frac{1}{10} \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{10^n}, \quad -10 < x < 10$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{10^{n+1}}, \quad -10 < x < 10$$

$$= \frac{1}{10} - \frac{x}{10^2} + \frac{x^2}{10^3} - \frac{x^3}{10^4} + \dots \quad \boxed{-10 < x < 10}$$

$$14 a) f(x) = \ln(1+x)$$

$$= \ln(1-(-x))$$

*see book example* →

$$= - \sum_{n=1}^{\infty} \frac{(-x)^n}{n} \quad |x| < 1$$

$$= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n} \quad -1 < x < 1.$$

$$b) f(x) = x \ln(1+x)$$

$$= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{n+1}}{n} \quad -1 < x < 1$$

$$c) f(x) = \ln(1+x^2)$$

$$= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n}}{n} \quad -1 < x < 1.$$

$$18. f(x) = \tan^{-1}(\frac{x}{3})$$

$$= \sum_{n=1}^{\infty} (-1)^n \frac{(\frac{x}{3})^{2n-1}}{2n-1}$$

*see book example* →

$$-1 < \frac{x}{3} < 1$$

$$= \sum_{n=1}^{\infty} (-1)^n \frac{x^{2n-1}}{3^{2n-1} \cdot (2n-1)}$$

$$\boxed{-3 < x < 3}$$

$$26. \int \tan^{-1}(x^2) dx$$

$$= \int \left[ \sum_{n=1}^{\infty} (-1)^n \frac{(x^2)^{2n-1}}{2n-1} \right] dx$$

$$= \int \sum_{n=1}^{\infty} (-1)^n \frac{x^{4n-2}}{2n-1} dx$$

$$= \sum_{n=1}^{\infty} (-1)^n \cdot \frac{1}{2n-1} \int x^{4n-2} dx$$

$$= \sum_{n=1}^{\infty} (-1)^n \cdot \frac{1}{2n-1} \cdot \frac{x^{4n-1}}{4n-1}$$

$$= \sum_{n=1}^{\infty} (-1)^n \frac{x^{4n-1}}{(2n-1)(4n-1)}$$

$$|x^2| < 1 \Rightarrow |x| < 1$$

$$So \Rightarrow \boxed{-1 < x < 1.}$$