12.6 : 20, 22, (24), 21

20. $\sum_{n=1}^{\infty} \frac{(-2)^n}{n^n}$ | extra

$$\lim_{n\to\infty} \sqrt[n]{\frac{2^n}{n^n}} = \lim_{n\to\infty} \frac{2}{n} = 0$$

Thus by the root test, the series is absolutely convergent

22. $\sum_{n=2}^{\infty} \left(\frac{-2n}{n+1}\right)^{5n}$

$$\lim_{n\to\infty} \sqrt[n]{\left|\frac{-2n}{n+1}\right|^{5n}}$$

$$= \lim_{n\to\infty} \left(\frac{2n}{n+1}\right)^5$$

$$= 2^5.$$

Thus by the root test, the series is divergent.

extra

24. $\sum_{n=2}^{\infty} \frac{n}{(\ln n)^n}$

$$\lim_{n\to\infty} \sqrt[n]{\frac{n}{(\ln n)^n}} = \lim_{n\to\infty} \frac{n^{1/n}}{\ln n}$$

$$\overset{L'H}{=} \lim_{n\to\infty} \frac{\frac{1}{n} n^{1/n-1}}{\frac{1}{n}}$$

$$= \lim_{n\to\infty} \frac{1}{n^{1-1/n}}$$

$$= 0.$$

Thus, by the root test the series is absolutely convergent

21. $\sum_{n=1}^{\infty} \left(\frac{n^2+1}{2n^2+1}\right)^n$

$$\lim_{n\to\infty} \left(\left(\frac{n^2+1}{2n^2+1}\right)^n\right)^{1/n}$$

$$= \lim_{n\to\infty} \frac{n^2+1}{2n^2+1} \cdot \frac{1/n^2}{1/n^2}$$

$$= \lim_{n\to\infty} \frac{1+1/n^2}{2+1/n^2}$$

$$= \frac{1}{2}.$$

Thus by the root test, the series is convergent.