$\underline{12.6}$: 4, 6, 8, 10, 12, 22

4. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{2^n}{n^4}$

$\displaystyle\lim_{n\to\infty} \left| \frac{(-1)^n 2^{n+1}}{(n+1)^4} \cdot \frac{n^4}{(-1)^{n-1} \cdot 2^n} \right|$

$\displaystyle = \lim_{n\to\infty} 2 \cdot \left(\frac{n}{n+1}\right)^4 = 2.$

By the Ratio Test, the series is divergent.

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6. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4}$

$\displaystyle\lim_{n\to\infty} \left| \frac{(-1)^{n+1}}{(n+1)^4} \cdot \frac{n^4}{(-1)^n} \right|$

$\displaystyle = \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^4 = 1.$

The Ratio test is inconclusive

Should have noted

$\displaystyle\sum_{n=1}^{\infty} \left| \frac{(-1)^n}{n^4} \right| = \sum_{n=1}^{\infty} \frac{1}{n^4}$ is

convergent, so the series is absolutely convergent.

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8. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{100^n}$

$\displaystyle\lim_{n\to\infty} \left| \frac{(n+1)!}{100^{n+1}} \cdot \frac{100^n}{n!} \right|$

$\displaystyle = \lim_{n\to\infty} \frac{n+1}{100} = \infty.$

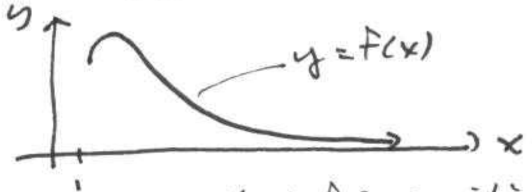
Thus by the Ratio test the series is divergent.

10. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n}{\sqrt{n^3+2}}$

$\displaystyle b_n = \frac{n}{\sqrt{n^3+2}}$

$\displaystyle\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{n/n^{3/2}}{\sqrt{1+2/n^3}} = 0$

$\displaystyle f(x) = \frac{x}{\sqrt{x^3+2}}$ has graph



We can see that f is positive and eventually decreasing, thus

$b_{n+1} \le b_n$ and $b_n > 0$

for all n. Thus by the alternating series test the series is convergent.

On the other hand, comparing

b_n to $\displaystyle a_n = \frac{n}{\sqrt{n^3}} = \frac{1}{\sqrt{n}}$ yields

$\displaystyle\lim_{n\to\infty} b_n/a_n = \lim_{n\to\infty} \frac{n^{3/2}}{\sqrt{n^3+2}} \cdot \frac{1/\sqrt{n^3}}{1/\sqrt{n^3}}$

$\displaystyle = \lim_{n\to\infty} \frac{1}{\sqrt{1+2/n^3}} = 1.$

Thus, since $\sum 1/\sqrt{n}$ is divergent so is $\displaystyle\sum_{n=1} b_n$, hence the series is conditionally convergent

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12. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin 4n}{4^n}$

$\displaystyle\left| \frac{\sin 4n}{4^n} \right| \le \frac{1}{4^n} = \left(\frac{1}{4}\right)^n$

By the comparison test, since $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n$ is convergence, so is $\displaystyle\sum_{n=1}^{\infty} \left| \frac{\sin 4n}{4^n} \right|$. Thus the series is absolutely convergent.