12.2: 22-32 even, 33, 36, 38.

22. $\lim_{n\to\infty} \frac{n+1}{2n-3} = \lim_{n\to\infty} \frac{1+1/n}{2-3/n} = \frac{1}{2}.$

Thus $\sum_{n=1} \frac{n+1}{2n-3}$ is divergent

24. $\sum_{k=1}^{\infty} \frac{k^2+2k}{k^2+6k+9}$,

$\lim_{k\to\infty} \frac{k^2+2k}{k^2+6k+9} = \lim_{k\to\infty} \frac{1+2/k}{1+6/k+9/k^2} = 1$

Thus $\sum_{k=1}^{\infty} \frac{k^2+2k}{k^2+6k+9}$ is divergent.

26. $\sum_{n=1}^{\infty} \frac{1+3^n}{2^n}$

$\lim_{n\to\infty} \left(\frac{1}{2^n} + \frac{3^n}{2^n}\right) = 0 + \lim_{n\to\infty} \left(\frac{3}{2}\right)^n = \infty$

Thus $\sum_{n=1}^{\infty} \frac{1+3^n}{2^n}$ is divergent

28. $\sum_{n=1}^{\infty} 0.8^{n-1} - \sum_{n=1}^{\infty} (0.3)^n$

$= \frac{1}{1-0.8} - \left(\sum_{n=1}^{\infty} (0.3)^{n-1} - 1\right)$

$= \frac{1}{\left(\frac{2}{10}\right)} - \frac{1}{1-0.3} + 1 = \frac{10}{2} - \frac{1}{\left(\frac{7}{10}\right)} + 1$

$= 5 - \frac{10}{7} + 1 = \frac{32}{7}.$

32. $\sum_{n=1}^{\infty} \frac{3}{5^n} + \sum_{n=1}^{\infty} \frac{2}{n}$

$= 3 \sum_{n=1}^{\infty} \left(\frac{1}{5}\right)^n + 2 \boxed{\sum_{n=1}^{\infty} \frac{1}{n}} = \infty$

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33. $\sum_{n=1}^{\infty} \left(\frac{1}{e}\right)^n + \sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

$= \left(\sum_{n=1}^{\infty} \left(\frac{1}{e}\right)^{n-1} - 1\right) + 1$

$= \frac{1}{1-e} - 1 + 1$

$= \frac{1}{1-e}.$

36. $\sum_{n=1}^{\infty} \frac{2}{(n+3)(n+1)}$

$\frac{2}{(n+1)(n+3)} = \frac{A}{n+1} + \frac{B}{n+3}$

$2 = An + 3A + Bn + B$

$= (A+B)n + (3A+B)$

$3A+B = 2 \Rightarrow B = 2-3A$

$A+B = 0 \Rightarrow A + 2 - 3A = 0$

$\Rightarrow A = 1,\ B = -1.$

$\sum_{n=1}^{\infty} \frac{2}{(n+3)(n+1)} = \sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+3}\right)$

$\frac{1}{2} - \cancel{\frac{1}{4}} + \frac{1}{3} - \cancel{\frac{1}{5}} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{6}} + \cancel{\frac{1}{5}} - \frac{1}{7} + \cancel{\frac{1}{6}} - \frac{1}{8} - \cdots \quad \left(= \frac{1}{2} + \frac{1}{3} = \frac{5}{6}\right)$

$= \sum_{n=1}^{\infty} \frac{1}{n+1} - \sum_{n=3}^{\infty} \frac{1}{n+1}$

$= \frac{1}{2} + \frac{1}{3} + \sum_{n=3}^{\infty} \frac{1}{n+1} - \sum_{n=3}^{\infty} \frac{1}{n+1} = \frac{5}{6}.$

38. $\sum_{n=1}^{\infty} \ln(n/(n+1)) = \sum_{n=1}^{\infty} (\ln(n) - \ln(n+1))$

$= \sum_{n=1}^{\infty} \ln n - \sum_{n=1} \ln(n+1)$

$= \sum^{\infty} \ln n - \sum^{\infty} \ln(n) = \ln(1) = 0.$