12.1 : 35, 40, 42, 48, 66 ~~[illegible]~~

35. see bottom of page

40. $\frac{0}{1+\sqrt{n}} \le \frac{|\sin 2n|}{1+\sqrt{n}} \le \frac{1}{1+\sqrt{n}}$

$\lim_{n\to\infty} \frac{1}{1+\sqrt{n}} = 0 = \lim_{n\to\infty} \frac{0}{1+\sqrt{n}}$

Thus, by the squeeze theorem

$\lim_{n\to\infty} \frac{\sin 2n}{1+\sqrt{n}} = 0.$

---

42. $\lim_{n\to\infty} \frac{(\ln n)^2}{n} \; \left(\frac{\infty}{\infty}\right)$

$\overset{L'H}{=} \lim_{n\to\infty} \frac{(2\ln n)/n}{1} = \lim_{n\to\infty} \frac{2\ln n}{n} \left(\frac{\infty}{\infty}\right)$

$\overset{L'H}{=} \lim_{n\to\infty} \frac{2}{n} = 0.$

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48. Just graph ~~[illegible]~~ $f(x) = \sqrt{x}\sin(\pi/\sqrt{x})$ on $0 \le x \le 100$ to see that the limit is approximately π.

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35. $a_n = \cos^2 n / 2^n$

$0 = \frac{0}{2^n} \le a_n \le 1/2^n$

~~[illegible]~~ $0 \le \lim_{n\to\infty} a_n \le \lim_{n\to\infty} \frac{1}{2^n} = 0$

$\Rightarrow \boxed{\lim_{n\to\infty} a_n = 0}.$

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66. $a_n = n + \frac{1}{n} \le n+1 \le n+1+\frac{1}{n+1} = a_{n+1}$

12.2 : 6, 10, 12-20 even

6. You ~~[illegible]~~ compute the partial sums to see that they converge to

$\sum_{n=1}^{\infty} \left(\frac{6}{10}\right)^{n-1} = \frac{1}{1-\frac{6}{10}} = \frac{1}{\frac{4}{10}} = \frac{5}{2}.$

---

10 a) they are the same

b) $\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots$

$\sum_{i=1}^{n} a_j = a_j + a_j + a_j + \cdots = n a_j$

They are not the same

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12. $\frac{1}{8} - \frac{1}{4} + \frac{1}{2} - 1 + \cdots$

$= \frac{1}{8}(1 - 2 + 4 - 8 + \cdots)$

$= \frac{1}{8}\sum_{n=1}^{\infty} (-2)^{n-1}$ $\boxed{\text{DIVERGENT}}$

---

14. $1 + \frac{4}{10} + \frac{16}{100} + \frac{64}{1000} + \cdots$

$= \sum_{n=1}^{\infty} \left(\frac{4}{10}\right)^{n-1} = \frac{1}{1-\frac{4}{10}} = \frac{10}{6} = \frac{5}{3}.$

---

16. $\sum_{n=1}^{\infty} \frac{10^n}{(-9)^{n-1}} = \sum_{n=1}^{\infty} \frac{10^{n-1}\cdot 10}{(-1)^{n-1} 9^{n-1}}$

$= 10 \sum_{n=1}^{\infty} (-1)^{n-1}\left(\frac{10}{9}\right)^{n-1}$ DIVERGENT

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18. $\sum_{n=0}^{\infty} \left(\frac{1}{\sqrt{2}}\right)^n = \sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{2}}\right)^{n-1}$

$= \frac{1}{1-\frac{1}{\sqrt{2}}} = \frac{1}{\frac{\sqrt{2}-1}{\sqrt{2}}} = \frac{\sqrt{2}}{\sqrt{2}-1}.$

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20. $\sum_{n=1}^{\infty} \frac{e^n}{3^{n-1}} = e\sum_{n=1}^{\infty}\left(\frac{e}{3}\right)^{n-1}$

$= e \cdot \frac{1}{1-\frac{e}{3}} = e \cdot \frac{1}{\frac{3-e}{3}} = \frac{3e}{3-e}.$