12.11 : 4, 6, 9, 10, 14(a)

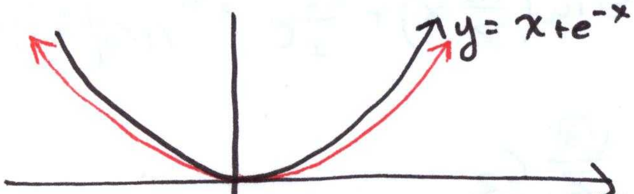
4. $f(x) = x + e^{-x}$

$$= x + \sum_{n=0}^{\infty} \frac{(-x)^n}{n!}$$

$$= x + \left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots\right)$$

$$= 1 + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots$$

$$T_3(x) = 1 + \frac{x^2}{2!} - \frac{x^3}{3!}$$



You plot on your calculator.

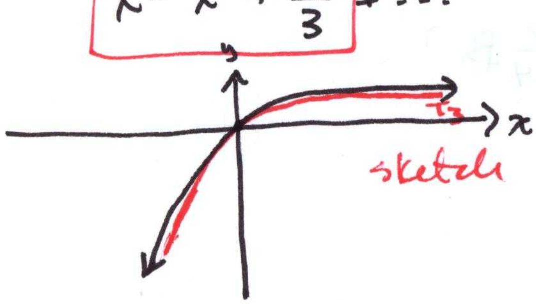
6. $f(x) = e^{-x}\sin x$

$$= \left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots\right) \cdot \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\right)$$

$$= x - x^2 - \frac{x^3}{3!} + \frac{x^3}{2!} + \frac{x^4}{3!} - \frac{x^4}{3!} + \cdots$$

$T_3(x)$

$$= \boxed{x - x^2 + \frac{x^3}{3}} + \cdots$$



$$T_k(x) = 1 - 2(x-1) + \cdots + (-1)^k (k+1)(x-1)^k.$$

9. $f(x) = x e^{-2x}$

$$= x\left(\sum_{n=0}^{\infty} \frac{(-2x)^n}{n!}\right)$$

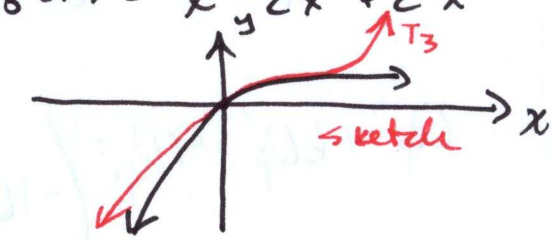
$$= x \sum_{n=0}^{\infty} (-1)^n \frac{2^n}{n!} x^n$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{2^n}{n!} x^{n+1}$$

$$= x - 2x^2 + \frac{4}{2!}x^3 - \cdots$$

$$= x - 2x^2 + 2x^3 - \cdots$$

$$T_3(x) = x - 2x^2 + 2x^3$$



14(a) $f(x) = x^{-2} \quad a = 1$

$f(1) = 1$

$f'(x) = -2x^{-3}, \quad f'(1) = -2 = -2!$

$f''(x) = 2\cdot 3\, x^{-4}, \quad f''(1) = 3!$

$f'''(x) = -2\cdot 3\cdot 4\, x^{-5}, \quad f'''(1) = -4!$

$$x^{-2} = f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \frac{f'''(1)}{3!}(x-1)^3 + \cdots$$

$$= 1 - 2!(x-1) + \frac{3!}{2!}(x-1)^2 - \frac{4!}{3!}(x-1)^3 + \cdots$$

$$= 1 - 2(x-1) + 3(x-1)^2 - 4(x-1)^3 + \cdots$$

$$= \sum_{n=0}^{\infty} (-1)^n (n+1)(x-1)^n$$

10. $f(x) = \tan^{-1} x$

$$f(1) = \tan^{-1}(1) = \frac{\pi}{4}$$

$$f'(x) = \frac{1}{1+x^2}, \quad \boxed{f'(1) = \frac{1}{2}}$$

$$f''(x) = -(1+x^2)^{-2}(2x)$$

$$= \frac{-2x}{(1+x^2)^2}, \quad f''(1) = \frac{-2}{4} = -\frac{1}{2}.$$

$$f'''(x) = \frac{(1+x^2)^2(-2) + 2x\left(2(1+x^2)\cdot 2x\right)}{(1+x^2)^4}$$

$$f'''(1) = \frac{-2\cdot(2)^2 + 2(2\cdot 2\cdot 2)}{2^4}$$

$$= \frac{16-8}{16} = \frac{1}{2}$$

$\vdots$

$$\tan^{-1}(x) = f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \frac{f'''(1)}{3!}(x-1)^3 + \cdots$$

$$= \frac{\pi}{4} + \frac{1}{2}(x-1) - \frac{1}{2}\cdot\frac{1}{2!}(x-1)^2 + \frac{1}{2}\cdot\frac{1}{6}(x-1)^3 + \cdots$$

$T_3(x)$

$$= \boxed{\frac{\pi}{4} + \frac{1}{2}(x-1) - \frac{1}{4}(x-1)^2 + \frac{1}{12}(x-1)^3} + \cdots$$