

Show all work to receive partial credit!

1. (10 pts) A spring-mass system with mass m , spring constant k , and damping constant c has an equation of motion satisfying

$$x'' + \frac{c}{m}x' + \frac{k}{m}x = 0, \quad x(0) = x_0, \quad x'(0) = v_0.$$

If $m = 1\text{kg}$, $k = 25\text{N/m}$, $c = 8\text{kg/s}$, $x_0 = 0\text{m}$, and $v_0 = 3\text{m/s}$, find the equation of motion.

1 pt

$$x'' + 8x' + 25x = 0, \quad x(0) = 0, \quad x'(0) = 3$$

χ_0 :

$$\lambda^2 + 8\lambda + 25 = 0 \quad \lambda = \frac{-8 \pm \sqrt{64 - 100}}{2} = -4 \pm 3i$$

Gen. Soln.

4 pt

$$x(t) = c_1 e^{-4t} \cos 3t + c_2 e^{-4t} \sin 3t$$

χ'

3 pt

$$x'(t) = -4c_1 e^{-4t} \cos 3t - 3c_1 e^{-4t} \sin 3t \\ - 4c_2 e^{-4t} \sin 3t + 3c_2 e^{-4t} \cos 3t$$

1 pt

$$0 = x(0) = c_1$$

$$\boxed{c_1 = 0}$$

$$3 = x'(0) = -4c_1 + 3c_2$$

$$\boxed{c_2 = 1}$$

1 pt

$$\text{So } x(t) = e^{-4t} \sin 3t$$