Math 158-QUIZ 6 NAME: Key

Show all work to receive partial credit!

(4 pts) 1. Find the general solution y_h of $y'' + y' + y = 0$.

2pts.

$$\lambda^2 + \lambda + 1 = 0 \qquad \lambda = \frac{-1 \pm \sqrt{1-4}}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$$

2pts.

$$y_h(x) = c_1 e^{-\frac{1}{2}x} \cos\left(\frac{\sqrt{3}}{2}x\right) + c_2 e^{-\frac{1}{2}x} \sin\left(\frac{\sqrt{3}}{2}x\right)$$

(6 pts) 2. Find a particular solution of $y'' + y' + y = \frac{3}{2}e^{2x}$ given guess $y_p(x) = Ae^{2x}$, and then write down the general solution using your answer in part 1.

4pts.

$$y_p' = 2Ae^{2x}$$

$$y_p'' = 4Ae^{2x}$$

$$y_p'' + y_p' + y_p = 7Ae^{2x} = \frac{3}{2}e^{2x} \implies 7A = \frac{3}{2}, \quad A = \frac{3}{14}$$

$$\boxed{\text{So } y_p(x) = \frac{3}{14}e^{2x}}$$

2pts

$$y(x) = c_1 e^{-\frac{1}{2}x} \cos\left(\frac{\sqrt{3}}{2}x\right) + c_2 e^{-\frac{1}{2}x} \sin\left(\frac{\sqrt{3}}{2}x\right) + \frac{3}{14}e^{2x}$$

(EC pts) 3. Using the general solution in part 2, find the solution of $y'' + y' + y = \frac{3}{2}e^{2x}$ subject to the initial conditions $y(0) = 0$, $y'(0) = 0$. Use the back of the page if you need to.

3pts [only if they get it all right or make a small mistake in which case +1

$$y'(x) = c_1\left(-\frac{1}{2}e^{-\frac{1}{2}x}\cos\left(\frac{\sqrt{3}}{2}x\right) - \frac{\sqrt{3}}{2}e^{-\frac{1}{2}x}\sin\left(\frac{\sqrt{3}}{2}x\right)\right) + c_2\left(-\frac{1}{2}e^{-\frac{1}{2}x}\sin\left(\frac{\sqrt{3}}{2}x\right) + \frac{\sqrt{3}}{2}e^{-\frac{1}{2}x}\cos\left(\frac{\sqrt{3}}{2}x\right)\right) + \frac{6}{14}e^{2x}$$

$$0 = y(0) = c_1 + \frac{3}{14} \implies c_1 = -\frac{3}{14}$$

$$0 = y'(0) = -\frac{1}{2}c_1 + \frac{\sqrt{3}}{2}c_2 + \frac{6}{14} \implies c_2 = \frac{2}{\sqrt{3}}\left(-\frac{6}{14} - \frac{3}{28}\right) = \frac{-30}{28\sqrt{3}}$$

→ back