

## Math 158-QUIZ 5

NAME: Key

Show all work to receive partial credit!

(3 pts) 1. Show that  $y(x) = xe^{2x}$  is a solution of  $y'' - 4y' + 4y = 0$ .

$$1 \text{ pt} \quad y' = e^{2x} + 2xe^{2x} = (1+2x)e^{2x}$$

$$1 \text{ pt} \quad y'' = 2e^{2x} + 2e^{2x} + 4xe^{2x} = (4+4x)e^{2x}$$

$$1 \text{ pt} \quad y'' - 4y' + 4y = (4+4x)e^{2x} - 4(1+2x)e^{2x} + 4xe^{2x} = 0.$$

(7 pts) 2. Find the solution of the differential equation

$$y'' + 5y' + 6y = 0, \quad y(0) = 1, \quad y'(0) = 1$$

by first finding the general solution (4pts), then finding  $c_1$  and  $c_2$  using the initial conditions (3pts).

$$4 \text{ pts.} \quad \left\{ \begin{array}{l} \lambda^2 + 5\lambda + 6 = 0 \quad (\lambda + 2)(\lambda + 3) = 0 \\ \lambda = -3, -2. \\ \boxed{y(x) = c_1 e^{-3x} + c_2 e^{-2x}} \quad ; \quad y'(x) = -3c_1 e^{-3x} - 2c_2 e^{-2x} \end{array} \right.$$

$$3 \text{ pts} \quad \left\{ \begin{array}{l} 1 = y(0) = c_1 + c_2 \\ 1 = y'(0) = -3c_1 - 2c_2 \end{array} \right.$$

$$c_1 = 1 - c_2$$

$$1 = -3(1 - c_2) - 2c_2 = -3 + c_2 \Rightarrow$$

$$\boxed{c_1 = -3}$$

$$\boxed{c_2 = 4}$$

$$\text{So } y(x) = -3e^{-3x} + 4e^{-2x}$$