Math 158-Quiz 10 **NAME:** Key

1. Use the *method of determinants* to find the general solution of the system

$$\begin{aligned} x' &= -2x + y, \quad x(0) = 1 && (1) \\ y' &= -3x + 2y, \quad y(0) = 0. && (2) \end{aligned}$$

Don't forget the initial conditions.

$$\begin{vmatrix} -2-\lambda & 1 \\ -3 & 2-\lambda \end{vmatrix} = (\lambda-2)(\lambda+2)+3 = \lambda^2 - 1. \quad \lambda = \pm 1$$

So $\boxed{x(t) = c_1 e^{-t} + c_2 e^{t}}$

$$y = x' + 2x$$
$$= -c_1 e^{-t} + c_2 e^{t} + 2c_1 e^{-t} + 2c_2 e^{t}$$

$$\boxed{y(t) = c_1 e^{-t} + 3c_2 e^{t}}$$

$1 = x(0) = c_1 + c_2 \quad \Rightarrow c_1 = 1 - c_2$

$0 = y(0) = c_1 + 3c_2 \quad \Rightarrow (1-c_2) + 3c_2 = 0 \Rightarrow c_2 = -\frac{1}{2}$

$2c_2 = -1. \quad \Rightarrow c_1 = \frac{3}{2}$

Thus
$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} \frac{3}{2}e^{-t} - \frac{1}{2}e^{t} \\ \frac{3}{2}e^{-t} - \frac{3}{2}e^{t} \end{pmatrix}$$

Extra Credit: From your answer above, write down two linearly independent solutions and then show that one of the them is a solution of the above system.

$x(t) = e^{-t}, \; y(t) = e^{-t}$

$x'(t) = -e^{-t}, \; y'(t) = -e^{-t}$

same

$-2x + y = -e^{-t}$

same

$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} e^{-t} \\ e^{-t} \end{pmatrix}, \begin{pmatrix} e^{t} \\ 3e^{t} \end{pmatrix}$ are solutions.