

7.1 1, 3, 5, 8, 10

Homework

1. $x_1 = x$, $x_2 = x_1'$. Then

$$x_1' = x_2$$

$$x_2' = -2x_2 - 3x_1$$

3. $x_1 = x$, $x_2 = x_1' = x'$, $x_3 = x_2' = x''$

$$x_1' = x_2$$

$$x_2' = x_3$$

$$x_3' = x_3 - (x_2)^2 + x_1^3 + t$$

5. $x_1 = x$, $x_2 = x_1' = x'$, $x_3 = x_2' = x''$

$$x_1' = x_2$$

$$x_2' = x_3$$

$$x_3' = -x_1 x_3 + x_2 x_1^4 + \sin t$$

8. $x_1 = x$, $x_2 = x_1' = x'$

$$y_1 = y, y_2 = y_1' = y'$$

$$z_1 = z, z_2 = z_1' = z'$$

$$x_1' = x_2$$

$$y_1' = y_2$$

$$z_1' = z_2$$



$$x_2' = \frac{1}{m} f(t, x_1, y_1, z_1)$$

$$y_2' = \frac{1}{m} g(t, x_1, y_1, z_1)$$

$$z_2' = \frac{1}{m} h(t, x_1, y_1, z_1)$$

$$10. \begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} c_1 x_1 + c_2 x_2 \\ c_1 y_1 + c_2 y_2 \end{pmatrix}'$$

$$= \begin{pmatrix} c_1 x_1' + c_2 x_2' \\ c_1 y_1' + c_2 y_2' \end{pmatrix}$$

$$= c_1 \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}' + c_2 \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}'$$

Since they are
solutions.

$$c_1 \begin{pmatrix} a_{11} x_1 + a_{12} y_1 \\ a_{21} x_1 + a_{22} y_1 \end{pmatrix} +$$

$$c_2 \begin{pmatrix} a_{11} x_2 + a_{12} y_2 \\ a_{21} x_2 + a_{22} y_2 \end{pmatrix}$$

$$= \begin{pmatrix} a_{11}(c_1 x_1 + c_2 x_2) & a_{12}(c_1 y_1 + c_2 y_2) \\ a_{21}(c_1 x_1 + c_2 x_2) & a_{22}(c_1 y_1 + c_2 y_2) \end{pmatrix}$$

$$= \begin{pmatrix} a_{11} x & a_{12} y \\ a_{21} x & a_{22} y \end{pmatrix}$$

$$\text{Thus } \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} c_1 x_1 + c_2 x_2 \\ c_1 y_1 + c_2 y_2 \end{pmatrix}$$

is a solution.