4.2: 1, 3, 5, ~~7~~

1. $x'' + \frac{c}{m}x' + \frac{k}{m}x = 0$, $x(0) = 1$, $x'(0) = 0$

$x'' + 20x' + 100x = 0$

$\lambda^2 + 20\lambda + 100 = 0$

$(\lambda+10)^2 = 0$, $\lambda = -10$

$x(t) = (c_1 + c_2 t)e^{-10t}$

$x'(t) = -10c_1 e^{-10t} + c_2 e^{-10t} - 10c_2 t e^{-10t}$

$1 = x(0) = c_1$

$0 = x'(0) = -10c_1 + c_2$

$\boxed{c_1 = 1 \quad c_2 = 10}$

So $x(t) = (1 + 10t)e^{-10t}$

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3. $x'' + \sqrt{5}x' + x = 0$, $x(0) = 3$, $x'(0) = 4$

$\lambda^2 + \sqrt{5}\lambda + 1 = 0$

$\lambda = \frac{-\sqrt{5} \pm \sqrt{5-4}}{2} = \frac{-\sqrt{5} \pm 1}{2}$

$\lambda_1 = -\frac{\sqrt{5}-1}{2}$, $\lambda_2 = -\frac{\sqrt{5}+1}{2}$

~~$x(t) = c_1 e^{\left(\frac{\sqrt{5}-1}{2}t\right)}$~~

$x(t) = c_1 e^{-\frac{\sqrt{5}+1}{2}t} + c_2 e^{-\frac{\sqrt{5}-1}{2}t}$

$x'(t) = -\frac{\sqrt{5}+1}{2}c_1 e^{-\frac{\sqrt{5}+1}{2}t} - \frac{\sqrt{5}-1}{2}c_2 e^{-\frac{\sqrt{5}-1}{2}t}$

nasty

$3 = x(0) = c_1 + c_2$

$4 = x'(0) = -\frac{\sqrt{5}+1}{2}c_1 + \frac{-\sqrt{5}-1}{2}c_2$

$c_1 = 3 - c_2$

$4 = -\frac{\sqrt{5}+1}{2}(3 - c_2) + \frac{-\sqrt{5}-1}{2}c_2$

$= -\frac{3\sqrt{5}+3}{2} + \frac{\sqrt{5}+1}{2}c_2 + \frac{-\sqrt{5}-1}{2}c_2$

$\Rightarrow c_2 = -\frac{3\sqrt{5}+3}{2} - 4$

$= \frac{-3\sqrt{5}-5}{2}$

$c_1 = \frac{6}{2} + \frac{3\sqrt{5}+5}{2} = \frac{3\sqrt{5}+11}{2}$

So $x(t) = \left(\frac{3\sqrt{5}+11}{2}\right)e^{-\frac{\sqrt{5}+1}{2}t} + \left(\frac{-3\sqrt{5}-5}{2}\right)e^{-\frac{\sqrt{5}-1}{2}t}$

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5. $x'' + 8x' + 25 = 0$, $x(0) = 0$, $x'(0) = 3$

$\lambda^2 + 8\lambda + 25 = 0$

$\lambda = \frac{-8 \pm \sqrt{64-100}}{2} = \frac{-8 \pm 6i}{2} = -4 \pm 3i$

$x(t) = c_1 e^{-4t}\cos 3t + c_2 e^{-4t}\sin 3t$

$x'(t) = c_1(-4e^{-4t}\cos 3t - 3e^{-4t}\sin 3t) + c_2(-4e^{-4t}\sin 3t + 3e^{-4t}\cos 3t)$

$0 = x(0) = c_1 \Rightarrow \boxed{c_1 = 0}$

$3 = x'(0) = -4c_1 + 3c_2 \Rightarrow \boxed{c_2 = 1}$

$x(t) = e^{-4t}\sin 3t$.

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7. skip it