3.4 1-11 odd: (158)

1. $y''+2y'+2y=0$

$\lambda^2+2\lambda+2=0$

$\lambda = \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm i$

$y(x) = c_1 e^{-x}\cos x + c_2 e^{-x}\sin x$

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3. $x''+x'+7x=0$

$\lambda^2+\lambda+7=0$

$\lambda = \frac{-1 \pm \sqrt{1-28}}{2} = \frac{-1 \pm \sqrt{9(-3)}}{2}$

$= -\frac{1}{2} \pm i\,\frac{3\sqrt{3}}{2}$

$x(t) = c_1 e^{-\frac{1}{2}x}\cos\left(\frac{3\sqrt{3}}{2}t\right) + c_2 e^{-\frac{1}{2}x}\sin\left(\frac{3\sqrt{3}}{2}t\right)$

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5. $x''+4x=0,\ x(\frac{\pi}{4})=1,\ x'(\frac{\pi}{4})=3$

$\lambda^2+4=0 \Rightarrow \lambda = \sqrt{-4} = \pm 2i$

$x(\theta) = c_1\cos 2\theta + c_2 \sin 2\theta$

$1 = x(\frac{\pi}{4}) = c_1\cos(\frac{\pi}{2}) + c_2\sin(\frac{\pi}{2})$

$= c_1 \cdot 0 + c_2 \cdot 1 = c_2$

$3 = x'(\pi/4) = -2c_1\sin(\frac{\pi}{2}) + 2c_2\cos(\frac{\pi}{2})$

$= -2c_1$

So $c_1 = -\frac{3}{2},\ c_2 = 1.$

and

$x(\theta) = -\frac{3}{2}\cos 2\theta + \sin 2\theta.$

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7. $y''+\frac{1}{4}y=0,\ y(\pi)=1,\ y'(\pi)=-1.$

$\lambda^2+\frac{1}{4}=0 \Rightarrow \lambda = \pm i\frac{1}{2}$

So $y(x) = c_1\cos(\frac{1}{2}x) + c_2\sin(\frac{1}{2}x)$

$1 = y(\pi) = c_2$

$-1 = y'(\pi) = -\frac{1}{2}c_1\sin(\frac{\pi}{2}) + \frac{1}{2}c_2\cos(\frac{\pi}{2})$

$= -\frac{1}{2}c_1$

So $c_1 = 2;\ c_2 = 1$ and

$y(x) = 2\cos(\frac{x}{2}) + \sin(\frac{x}{2}).$

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9. $y''+2y+5y=0;\ \lambda^2+2\lambda+5=0$

$\lambda = \frac{1}{2}(-2 \pm \sqrt{4-20}) = \frac{1}{2}(-2 \pm 4i)$

$= -1 \pm 2i$

So $y(x) = c_1 e^{-x}\cos 2x + c_2 e^{-x}\sin 2x$

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11. $y''+2y'+2y=0,\ y(\pi)=e^{-\pi},\ y'(\pi)=-2e^{-\pi}$

$\lambda^2+2\lambda+2=0$ (see #1)

$y(x) = c_1 e^{-x}\cos x + c_2 e^{-x}\sin x$

$y'(x) = -c_1 e^{-x}\cos x - c_1 e^{-x}\sin x - c_2 e^{-x}\sin x + c_2 e^{-x}\cos x$

$e^{-\pi} = y(\pi) = c_1 e^{-\pi}\cos\pi + c_2 e^{-\pi}\sin\pi$ ($\cos\pi \to -1$, $\sin\pi \to 0$)

$= c_1 e^{-\pi} \Rightarrow \boxed{c_1 = 1}$

$-2e^{-\pi} = y'(\pi) = -c_1 e^{-\pi}(-1) + c_2 e^{-\pi}(-1)$

$\Rightarrow -2 = c_1 - c_2$

$\Rightarrow c_2 = c_1 + 2 = 1.$

So $y(x) = -e^{-x}\cos x + e^{-x}\sin x$