HW: 3.3 1-13 odd

1. $y''-4y=0$

$\lambda^2-4=0 \Rightarrow \lambda=\pm 2$

Solns: $y(x)=e^{2x}, e^{-2x}$

General solution:

$$y(x)=c_1e^{2x}+c_2e^{-2x}.$$

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3. $y''-3y'+2y=0$

$\lambda^2-3\lambda+2=0$

$(\lambda-2)(\lambda-1)=0 \Rightarrow \lambda=1,2$

General solution:

$$y(x)=c_1e^{x}+c_2e^{2x}.$$

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5. $4y''+20y'+25y$, $y(0)=1$, $y'(0)=2$

$4\lambda^2+20\lambda+25=0$

$\lambda=\dfrac{-20\pm\sqrt{400-400}}{8}=-\dfrac{5}{2}$

Solutions: $e^{-5/2\,x}$, $xe^{-\frac{5}{2}x}$

So the general solution is

$$y(x)=c_1e^{-5/2x}+c_2xe^{-5/2x}$$

$1=y(0)=c_1$.

$2=y'(0)=\left[c_1\left(-\frac{5}{2}\right)e^{-5/2x}+c_2e^{-5/2x}+c_2\left(-\frac{5}{2}\right)xe^{-5/2x}\right]_{x=0}$

$=-\frac{5}{2}c_1+c_2$

$\boxed{c_1=1,}$ $c_2=2+\frac{5}{2}(1)$; $\boxed{c_2=\frac{9}{2}}$

So $y(x)=e^{-5/2x}+\frac{9}{2}xe^{-5/2x}$.

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7. $y''-y'-6y=0$, $y(0)=-1$, $y'(0)=1$

$\lambda^2-\lambda-6=0$

$(\lambda-3)(\lambda+2)=0$, $\lambda=-2,3$

So $y(x)=c_1e^{-2x}+c_2e^{3x}$

$-1=y(0)=c_1+c_2 \Rightarrow c_1=-1-c_2$

$1=y'(0)=-2c_1+3c_2$

$\hookrightarrow 1=+2+2c_2+3c_2$

$\Rightarrow 5c_2=-1$

$c_2=-1/5$, $c_1=-1+1/5=-4/5$

So $y(x)=-\frac{4}{5}e^{-2x}-\frac{1}{5}e^{3x}$

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9. $y''-5y'=0$

$\lambda^2-5\lambda=0$, $\lambda(\lambda-5)=0$

$\lambda=0,5$

$$y(x)=c_1+c_2e^{5x}.$$

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11. $y''+2\pi y'+\pi^2y=0$

$\lambda^2+2\pi\lambda+\pi^2=0$

$\lambda=\dfrac{-2\pi\pm\sqrt{4\pi^2-4\pi^2}}{2}=-\pi$

$$y(x)=c_1e^{-\pi x}+c_2xe^{-\pi x}.$$

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13. $z''-2z'-15z=0$, $\lambda^2-2\lambda-15=0$

$(\lambda-5)(\lambda+3)=0$, $\lambda=-3,5$

$$z(x)=c_1e^{-3x}+c_2e^{5x}.$$