3.1: 12, 13, 27, 29, 33 158

12. Since $-3y_1 = y_2$, y_1 and y_2 are linearly dependent.

13. Since $y_1 \neq c y_2$ for all numbers c, y_1 and y_2 are linearly independent.

27. $y_1 = 1 \Rightarrow y_1'' = y_1' = 0$

So $x y_1'' - y_1' = x \cdot 0 - 0 = 0$

So y_1 is a solution.

$y_2(x) = x^2$

$y_2'' = 2$, $y_2' = 2x$

So $x y_2'' - y_2' = 2x - 2x$
$= 0$

So y_2 is a solution.

Thus the general solution is

$$y(x) = c_1 y_1(x) + c_2 y_2(x) = c_1 + c_2 x^2.$$

29. $y_1(x) = \sin 3x$

$y_1'(x) = 3\cos 3x$

$y_1''(x) = -9\sin 3x$

$y_1'' + 9y_1 = -9\sin 3x + 9\sin 3x = 0$

So y_1 is a solution

$y_2(x) = \cos 3x$

$y_2'(x) = -3\sin 3x$

$y_2''(x) = -9\cos 3x$

$y_2'' + 9y_2 = -9\cos 3x + 9\cos 3x = 0$

So y_2 is a solution

y_1 & y_2 are linearly independent

thus the general solution is

$$y(x) = c_1 \sin 3x + c_2 \cos 3x$$

33. $y_1(x) = e^{3x}$

$y_1'(x) = 3e^{3x}$

$y_1''(x) = 9e^{3x}$

$y_1'' - 6y_1' + 9y_1 = 9e^{3x} - 18e^{3x} + 9e^{3x}$

So y_1 is a soln. $= 0$

$y_2(x) = xe^{3x}$

$y_2'(x) = e^{3x} + 3xe^{3x}$

$y_2''(x) = 3e^{3x} + 3e^{3x} + 9xe^{3x}$
$= 6e^{3x} + 9xe^{3x}$

$$y_2'' - 6y_2' + 9y_2 = \cancel{6e^{3x}} + \cancel{9xe^{3x}} - \cancel{6e^{3x}} - \cancel{18xe^{3x}} + \cancel{9xe^{3x}} = 0$$

So y_2 is a solution. Thus the general solution is

$$y(x) = c_1 e^{3x} + c_2 x e^{3x}.$$