

HW 2.1: 1, 2, 3, 6 from text

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$$1. \frac{dy}{dx} = xe^y$$

$$\Rightarrow \frac{dy}{e^y} = x dx$$

$$\Rightarrow \int e^{-y} dy = \int x dx$$

$$u = -y \Rightarrow \frac{du}{dy} = -1 \Rightarrow dy = -du$$

$$\Rightarrow -\int e^u du = \int x dx$$

$$-e^u = \frac{x^2}{2} + C$$

$$u = \ln e^u = \ln\left(-\frac{x^2}{2} - C\right)$$

$$\text{So } y(x) = -\ln\left(-\frac{x^2}{2} + C\right).$$

$$2. x \frac{dy}{dx} = 3y, y(2) = 5$$

$$\int \frac{dy}{y} = \int \frac{3}{x} dx, x, y \neq 0$$

$$\ln|y| = 3 \ln|x| + C$$

$$|y| = e^{\ln|x|^3 + C}$$

$$y(x) = \pm e^C |x|^3$$

$$\text{So } y(x) = C|x|^3$$

$$5 = y(2) = C \cdot 2^3 = 8C$$

$$\Rightarrow C = 5/8. \text{ So } y(x) = \frac{5}{8}|x|^3.$$

$$3. \frac{dP}{dQ} = 2\sqrt{P}$$

$$\Rightarrow \frac{dP}{\sqrt{P}} = 2dQ$$

$$\Rightarrow \int P^{-1/2} dP = 2 \int dQ$$

$$\Rightarrow 2P^{1/2} = 2Q + C$$

$$\Rightarrow (P^{1/2}(Q))^2 = (Q + \frac{C}{2})^2$$

$$\Rightarrow P(Q) = (Q + C)^2.$$

$$6. \frac{dr}{ds} = s r e^{s^2}, r(0) = 1$$

$$\int \frac{dr}{r} = \int s e^{s^2} ds$$

$$u = s^2, \frac{du}{ds} = 2s \Rightarrow \frac{1}{2} du = s ds$$

$$\int \frac{dr}{r} = \frac{1}{2} \int e^u du$$

$$\ln|r| = \frac{1}{2} e^u + C$$

$$|r| = e^{\frac{1}{2} e^{s^2} + C}$$

$$r(s) = \pm e^C e^{\frac{1}{2} e^{s^2}}$$
$$= C e^{\frac{1}{2} e^{s^2}}$$

$$1 = r(0) = C e^{\frac{1}{2} \cdot 1} = C e^{\frac{1}{2}}$$

$$\text{So } C = e^{-\frac{1}{2}} \text{ and}$$

$$r(s) = e^{-\frac{1}{2}} e^{\frac{1}{2} e^{s^2}}$$
$$= e^{\frac{1}{2} e^{s^2} - \frac{1}{2}}$$