Some reflections on mathematics and mathematicians. Simple questions, complex answers

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… *"The Mathematic does not have own existence. It is only an arbitrary code, designed to describe physical observations or philosophical concepts. Each can adapt it to its own needs."*
Dr. John Keyser, Ph.D. in Physics[2]

SUMMARY

In this work we present some reflections on mathematics and mathematicians. Special emphasis is placed on the questions (1) what is mathematics? And (2) what is a mathematician? Some reflections and open questions are posed at the end of the work.

0. Introduction.

Professions have played a key role in the development of disciplinarily, and vice versa. Within some disciplines the direct binding to a profession or a field have over time been loosened and (re)searching knowledge for its own sake has become a main driving force of a new, advanced kind of disciplinarily. For mathematics these historical shifts are symptomatic in the debates over the discipline's *true nature*. While the relationship between science, technology and mathematics historically the last 200 years has been rather symbiotic, mathematics today serve so many different professions and fields, that a unified, valid definition of its *nature* is hard to find.

Mathematical discoveries have come both from the attempt to describe the natural world and from the desire to arrive at a form of inescapable truth from careful reasoning. These remain fruitful and important motivations for mathematical thinking, but in the last century mathematics has been successfully applied to many other aspects of the human world: voting trends in politics, the dating of ancient artifacts, the analysis of automobile traffic patterns, and long-term strategies for the sustainable harvest of deciduous forests, to mention a few. Today, mathematics as a mode of thought and expression is more valuable than ever before. Learning to think in mathematical terms is an essential part of becoming a liberally educated person.

Much of mathematics is itself about mathematical objects. This is part of why mathematics can seem like an arcane and up-approachable field to an outsider. Fortunately in asking *What is Mathematics?* We are asking about the meaning and consequences of Mathematics as

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[2] Isaac Asimov-**"The Red Queen's Race"**, in Astounding Science-Fiction, January 1949. Reprinted in **"The Complete Stories II"**, Doubleday, 1990.

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connected to the larger world, that is, we are asking what Mathematics means outside of its own world, in answering our question we can largely ignore many of the details of Mathematics[3].

Plato tries to clarify a position when indicating that the mathematical objects have their own existence, beyond the mind.Aristotle saw the mathematics like one of the divisions of the knowledge that was different from the physical knowledge and the theological one. He denied that the mathematics were a theory of an external knowledge, independent and unnoticeable. It associated to the mathematics with a reality where the knowledge obtains by experimentation, observation and abstraction. This position joint party that the construction of the mathematical ideas occurs through idealizations realized by the mathematicians like a result of its experience with objects in a specific context.The points of view of Plato and Aristotle have represented the great poles where the discussion has oscillated about the nature of the mathematics.

But the absolutist vision entered crisis due to the discovery of some contradictions found in certain theorems that comprised of mathematical systems considered rigorous. For example, Russell demonstrated that the logical system of Frege was inconsistent. The paradox of the property of *being an element of itself* (a set is element of itself if and only if it is not element of itself) did collapsed its law number 15. But the mathematic one is certain and if all theorems are true, how can contradictions exist between their theorems? Something must be mistaken in the foundation of the mathematics. Paul Ernest proposes a socio-constructivist vision of the mathematical one[4]. In this vision it is considered that the mathematical truth is fallible and correctable, and that is the overhaul always open. This theory takes from the conventionalism the idea that the human language with their rules and agreements plays an important role in the establishment and justification of the mathematical truths. Also he takes from quasi-empiricism, its epistemology of the fallibility of the mathematics and the principle of which the mathematical concepts and knowledge change by means of conjectures process and refutations.
Mathematics is the subject where answers can definitely be marked right or wrong, either in the classroom or at the research level. Mathematics is the subject where statements are capable in principle of being proved or disproved, and where proof or disproof bring unanimous agreement by all qualified experts—all who understand the concepts and methods involved.

Reasoning about mental objects (concepts, ideas) that compels assent (on the part of everyone who understands the concepts involved) is what we call "mathematical". This is what is meant by *mathematical certainty*. It does not imply infallibility![5]

History shows that the concepts about which we reason with such conviction have sometimes surprised us on closer acquaintance, and forced us to re-examine and improve our reasoning.

Ah, but on the library shelves, in the math section, all those formulas and proofs, isn't that math? No, as long as it just sits on the shelf, it's just ink on paper. It becomes

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[3] An interesting review of various postures on the nature of Mathematics may be found in **"Lectures on the Foundations of Mathematics"** of John L. Bell.

[4] Ernest, P. (1991)-**"The Philosophy of Mathematics"**, London, The Falmer Press, also cf. Morris Kline (1980)-"**Mathematics: The Loss of Certainty"**, Oxford University Press.

[5] http://www.math.unm.edu/~rhersh/Definition%20of%20mathematics.doc

mathematics; it comes alive, when somebody starts to read it. And of course, it was alive when it was being thought and written by some mathematician.

Mathematical conclusions are decisive. Just as physical or chemical knowledge can be independently verified by any competent experimenter, an algebraic or geometric proof can be checked and recognized as a proof by any competent algebraist or geometer. Saunders MacLane, among others, said, *"What characterizes mathematics is that it's precise"*. But what, precisely, should be meant here, by *precise*? Not *numerical* precision. A huge part of modern mathematics, including MacLane's contribution, is geometrical or syntactical, not numerical. Should *precise* mean formally explicit, expressed in a formal symbolism? No. There are famous examples in mathematics of conclusive *visual* reasoning, accepted as *mathematical proof* prior to any *post hoc* formalization. Several famous mathematicians have said *"You don't really understand a mathematical concept until you can explain it to the first person you meet in the street"*.

Probably the correct interpretation of *precise* should be simply, *subject to conclusive, irrefutable reasoning*. So I am accepting the familiar claim, *"Mathematics is characterized above all by precision"*, but only after *unpacking* what we should mean by *precise*.

In the past 25 or 30 years, it has come to be recognized that mathematics is not a fixed, unitary, absolute body of knowledge that changes only by growth at the periphery. Advances in the history and philosophy of mathematics, the sociology of knowledge, and post-modernist thought[6] have shown that the myth of the unchanging nature of mathematics is probably held in place by the use of single term *mathematics* for several diverse domains of knowledge and discursive practice. School mathematics and the research mathematician's pure mathematics are wholly different areas of study. Controversy has erupted over the natures of both of these domains: the first is the subject of political contestation; the latter of philosophical dispute[7].

In this work we present some reflections on mathematics and mathematicians motivated by a recent work of Pan Shengliang[8]. Special emphasis is placed on the questions what is mathematics? And what is a mathematician? Some open questions related to these topics are posed at the end of the work.

1. What is mathematics?

One of the oldest of all fields of study is that now known as Mathematics. Often referred to, used, praised, and disparaged, it has long been one of the most central components of human thought, yet how many of us could describe what mathematics really is?

Searching with Google for "definitions of mathematics" gives approximately 8.620.000 hits[9]. From a quite traditional and very general view mathematics is often seen as (...) *"a science (or group of related sciences) dealing with the logic of quantity and shape and arrangement"*[10]. However such a characterisation only describes what, not how (or why). Hence methodological aspects that might be of significance are not mentioned. A description that combines what and how (underlined in the quote by me) is found in Wikipedia where mathematics is seen as (...) *"the body of knowledge centred on concepts such as quantity, structure, space, and change, and also the academic discipline that studies them"*. Benjamin Peirce called it *"the science that draws necessary conclusions"*. Other practitioners of mathematics maintain that

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[6] See for example Ernest, P. Ed. (1994)-**"Mathematics, Education and Philosophy: An International Perspective"**, London, The Falmer Press.

[7] Ernest, P. (1991)-Op. Cit.

[8] Shengliang, P. (2003)-**"Some reflections on mathematics, mathematics education and mathematicians"**, The China Papers, July, 95-99.

[9] November 2009.

[10] http://www.thefreedictionary.com/science

mathematics is the science of pattern, that mathematicians seek out patterns whether found in numbers, space, science, computers, imaginary abstractions, or elsewhere. Mathematicians explore such concepts, aiming to formulate new conjectures and establish their truth by rigorous deduction from appropriately chosen axioms and definitions[11].
The common people belief among many students is that Mathematics is about numbers, formulas and cranking out computations. It is the unconsciously held delusion that Mathematics is a set of rules and formulas that have been worked out by God knows who for God knows why, and the student's duty is to memorize all this stuff. This position can take to diverse mistaken answers to the question that heads this section.

Kasner and Newman's point of view is that, *"Mathematics is the science which uses easy words for hard ideas"*[12]. According to Kant, *"the science of mathematics presents the most brilliant example of how pure reason may successfully enlarge its domain without the aid of experience"*.
Said Feynman *"To those who do not know Mathematics it is difficult to get across a real feeling as to the beauty, the deepest beauty of nature. ... If you want to learn about nature, to appreciate nature, it is necessary to understand the language that she speaks in"*.

In fact, like other sciences, mathematics reflects the laws of the material world around us and serves as a powerful instructional tool for understanding Nature. Mathematics reveals the hidden patterns that empower us to understand better the information-laden world in which we live. As a science of abstract objects, Mathematics relies on logic rather than on observation for the purpose of stating truths, yet employs observation, simulation, and even experimentation as a means of discovering truth. Through its results, mathematics offers science both a foundation of truth and a standard of certainty. Mathematics offers distinctive modes of thought which are both versatile and powerful; including modeling, abstraction, optimization, logical analysis, inference from data, and use of symbols. Mathematics enables us to read critically, to identify fallacies, to detect bias, to assess risk, and to suggest alternatives. The resolution of mathematical problems supplies people with techniques, which can be used in different areas; even to everyday problems mathematical thinking is logical and strict, intuitive and creative, dynamic and changing.
By other hands, an opposite position is due to Russell *"Mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true"*. It is clear that the definition of Russell does not help us much.

When mathematics is understood in the broadest sense, not overstepping the thresholds to neighbouring academic disciplines, the field embraces 97 different specific kinds or sub-branches of mathematics according to MSC (of which for instance ordinary differential equations is just one)[13].

In a principle inquiry of definitions of mathematics Bonnie Gold identifies and discusses critically nine major claims[14]. As a result of the inspection she outlines 13 criteria for *good definitions*. Taken collectively these criteria seem to have a dual function, to describe (valid) internal cohesions within the discipline of mathematics and to relate what one could call

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[11] http://en.wikipedia.org/wiki/Mathematics#Mathematics_and_physical_realityA

[12] See Blank, B. E. (2001)-**"What is mathematics? An elementary approach to ideas and methods"**, Notices of the AMS, December, 1325-1329; a review of the classic book of Richard Courant and Herbert Robbins, Oxford University Press, USA; 2 edition (July 18, 1996).

[13] Rusin, D. (2004)-**"The Mathematical Atlas. A gateway to modern mathematics"**, in http://www.math-atlas.org/welcome.html, http://www.math.niu.edu/~rusin/known-math/index/tour.html and http://www.math.niu.edu/~rusin/known-math/index/tour.html

[14] Gold, B. (2003)-**"What is mathematics? I: The question"** Monmouth University, in http://www.math.utep.edu/Faculty/pmdelgado2/Math1319/Philosophy/Bonnie.doc .

mathematically to other disciplinarities. These two concerns are of course often closely related. Of the nine types of descriptions of mathematics there are hardly any that does not play some role in other disciplines. It is therefore not likely to find one *single* aspect that makes mathematics unique, and which can be used solely to define every former, present and future kind of mathematics.

As pointed to above a philosophical challenge for mathematics is that during its historical purification process, becoming an academic discipline, it tends to obliterate its own foundations. At the heart of the discipline as *established* there seems to be a kind of safety-game where a *universal given's* of mathematics makes a critical questioning of the discipline irrelevant and inadequate. This intellectual *laziness* (or this sensible pragmatic taken for granted attitude) is transmitted to mathematics education because mathematics of course here normally is based on and focuses the stability and not the slow development of the discipline. This tendency consolidates the idea that mathematics is given rather than developed and thus may function as another set of blinkers for how disciplinarily is generated.

Gold dismisses the claim that *mathematics is what mathematicians do*. Although she admits that one (...) *could modify it by saying that it is what mathematicians do when acting as mathematicians,* she doubts that one can avoid circularity when specifying what it is to act as a mathematician. However if one looks at this definition in the light of pragmatics (which Gold does not), it could be further refined. Mathematics as discipline could be described by the full set of practical and intellectual acts that are at work when doing mathematics (but not only). In other words, even mathematics needs to be seen, not just as products, but as processes. This will obviously accumulate into a long list, at least containing activities such as theorising, doing inductions and deductions, defining, arguing, calculating, giving premises, concluding, etc. This implies a *pragmatic* understanding of language and communication.
In discussions there at this point often tends to appear an opposition between applied and pure mathematics[15], where the kind of acts related to these types of doing mathematics are said to be qualitatively different (cf. paragraph C in Gold's paper). In any case the question of which mental and practical activities that are involved can not be finalised without a valid description of the content of mathematics (to the degree this is practically and principally possible). Gold finds that listing sub-fields is the most common way of defining mathematics.

Even if this gives some kind of concreteness to the question there are several dangers: (...) *"such definitions risk becoming dated by the evolution of mathematics; even if we make our list include all the current Mathematics Reviews subject classifications, new subjects are being added all the time. Second, they emphasize the separateness of the different branches of mathematics, whereas if there has been any lesson from the development of mathematics in the last 50 years, it is the unity of mathematics, the complex web of interconnections between the supposedly different fields, even those which seem to have very different flavors (more on this in section IV). Third, they give no assistance in recognizing a new kind of mathematics when it appears"*[16].

In other words, from our perspective one should combine a *synchronic* and a *diachronic* view of the discipline, a conclusion which of course is close to the former that one needs to differentiate between *products* and *processes*. Nevertheless it takes into account the interplay between *stability* and *dynamics*.

[15] See Stewart, I.-**"Letters to a young mathematician"**, Basic Books, 2006; mainly the Letter 15 "Pure or Applied".
[16] Gold, B. (2003)-Op. Cit., p.4.

Gold further claims that the difficulty with (...) *"finding a common subject has caused people to turn to the methodology of mathematics to find its unifying theme, mathematics being unique among the sciences in making deductions from axioms the cornerstone of its reasoning"*[17]. The crucial role of axioms in mathematics is agreed upon in mathematics. Mathematics is built and continues to be built upon this particular genre. Metaphorically an axiom functions as a humming top in a supposedly eternal spin, so that it will never fall. From a (pragmatic) speech act perspective it can simplistically be described by an utterance beginning with *Given that*... It is the final preciseness, creativity and relevance of the description of the set of axioms that will bring mathematics further, closer to the cutting edge of its disciplinarily. But it is by the same token the continuous growth of (interrelated) axioms that makes mathematics stable. Paradoxically, using language to create a fixed point of departure is also what gives mathematics the imaginative freedom and makes *pure* mathematics possible (and even free, fresh and fascinating). According to Gold, Nevanlinna expresses a similar sentiment *"Mathematics combines two opposites, exactitude and freedom"*[18].

Surprisingly, and for some, provokingly, this view makes language and mathematics to rather inherited (semiotic) phenomena. Hence while language in general and fiction in particular can be seen (with Umberto Eco) as the tool with which one in principle *can* lye, the regime of axioms in mathematics leads to the opposite, a position which is at the heart of Benjamin Peirce's famous definition of mathematics as the science which draws necessary conclusions.

In this perspective one of the foundations of mathematics is a purification of a particular kind of speech act where lying is made impossible. You can make mistakes, but not lye, once given the axioms that close the mathematical entities. Consequently, if you are lying or cheating deliberately, what you do is not (according to) mathematics. The main reason for that this is possible is the axiomatic closing of open signs. According to semiotic theory signs in *natural* language are under the law of semiotic, a never-ending growth in the meaning of all concepts over time. In mathematics however such concepts/objects can not be part of an axiomatic act/definition.

How said before, one popular definition of mathematics is the discipline that studies *patterns*. Gold argues that this view does not distinguish structures found in mathematics from other structures[19]. Mathematics is for instance not interested in the patterns of atoms or molecules, rather, (...) *"mathematics is concerned with the properties of patterns, the general relationships between patterns, how they behave, and so on"*. To see mathematics as the science of patterns implies a structuralist perspective[20]. Reuben Hersh, famous for advocating the (implicit pragmatic) view that mathematics is what mathematicians do, writes critically in

"What Is Mathematics, Really?".

The definition, *science of patterns* is appealing[21]. It's closer to the mark than *"the science that draws necessary conclusions"* (Benjamin Peirce), *"the study of form and quantity"*[22] or *"The*

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[17] Idem.

[18] See page 456 of Nevanlinna, R. (1966)-**"Reform in Teaching Mathematics"**, *Monthly*, 73: 451-464.

[19] Gold, B. (2003)-Op. Cit.

[20] Cf. **"Taming the infinite. The story of Mathematics"** of Ian Stewart, published by Quercus Publishing PLC, UK, 2007 and Devlin, K. J. (2000)-**"The language of Mathematics: making the invisible visible"**, W. H. Freeman and Company.

[21] See Letter 3 of Stewart-Op.Cit.

[22] Webster's Unabridged Dictionary. Also cf. the page http://www.mathacademy.com/pr/quotes/index.asp?ACTION=AUT&VAL=Steen or the Lynn Arthur Steen's Home Page http://www.stolaf.edu/people/steen/

mathematics is the study of the true thing of the hypothetical situations" (Charles S. Peirce)[23]. This one is its essence and its definition. Unlike formalism, structuralism allows mathematics a subject matter. Unlike Platonism, it doesn't rely on a transcendental abstract reality. Structuralism grants mathematics unlimited generality and applicability. Structuralism is valid as a partial description of mathematics, an illuminating comment. As a complete description, it's unsatisfactory[24].

The Marxist case. Many Marxist historians maintain the Engels' definition *"Pure mathematics deals with the space forms and quantity relations of the real world -that is, with material which is very real indeed"*[25] and they continue insisting on the existence of objective laws of the development of the mathematics, without showing which these laws are and how they work to predict its development.

Kolmogorov[26] in a very famous paper for the Marxists says *"In the continuing relationship with the requirement of the technical and scientific knowledge, the wealth of quantitative relationships and forms space studied by the Mathematics, is constantly expands, so the general definition of Mathematics is filled with a content increasingly rich."*[27]
"In conclusion", says Sánchez, "*the definition of the object of the Mathematics given by Engels, continues being actual"*[28].
These Marxist philosophers, have created what I call a *Metaphilosophy of the Mathematics*, discussing and analyzing questions of the "classics of Marxism" (Marx, Engels and Lenin of course¡¡¡¡), and the social practice as the bases of the development of Science, underestimating other causes and factors systematically and, what is worse, subordinating mathematics researches to certain ideological goals and placed under a strict ideological scrutiny.[29]
On the matter they are sufficient the following affirmations[30]:
"As was the case with all the work of the classics, the Manuscript Mathematicians from Karl Marx, were a need for its general plan to fight." …

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23 "The Essence of Mathematics", ch. 3 of his unpublished **"Minute Logic"** online in http://www.unav.es/gep/EssenceMathematics.html (Spanish).
24 Hersh, R. (1997)-**"What Is Mathematics, Really?"**, Oxford: Oxford University Press and Yiparaki, O. (1999)-**"Another General Book on Mathematics?"**, Complexity, Vol. 4/4, pp 55-60.
25 Engels, F. (1975)-**"Anti-Duhring"**, La Habana, Editorial Pueblo y Educación (Spanish) available online in http://www.marxists.org/archive/marx/works/1877/anti-duhring/ch01.htm
26 See Vucinich, A. (2000)-**"Soviet Mathematics and Dialectics in the Stalin Era"**, Historia Mathematica, Vol. 27, Nº 1, 54-76 and Vucinich, A. (2000)-**"Soviet Mathematics and Dialectics in the Post-Stalin Era: New Horizons"**, Historia Mathematica, Vol. 29, Nº 1, 13-39, for a characterization of the soviet Mathematic.
27 See the Kolmogorov's article "Mathematics" of Bolshaya Sovietskaya Entsiklopedja, Great Soviet Encyclopedia (1936) online in http://www.kolmogorov.pms.ru/bse-mathimatic.html (Russian)
28 Sánchez F., C. (1987)-**"Conferences on philosophical and methodological problems of the Mathematic"**, Universidad de la Habana (Spanish).
29 See, for example, Alexandrov, A. D. (1964)-"Mathematics", in **"Philosophical Encyclopedia"**, t. III, p. 329, Sovietskaia Enziklopedia, Moskva (Russian); Alexandrov, A. D., A. N. Kolmogorov, M. A. Lavrentiev (eds) (1999)-**"The mathematics: its content, methods and meaning"**, Dover; Alekxeev, B. T.-"Dialectic of the mathematic knowledge" in F. B. Konstamtinov (1983)-**"Materialist Dialectic"**, Thought Editorial, T. 3 (Russian); Arkadi, U. (1981)-**"The dialectic and the methods scientific generals of investigation"**, La Habana, Social Sciences Editorial, pp.190-191; Casanovas, G. (1965)-**"The Mathematic and the Dialectic Materialism"**, La Habana, National Editorial of Cuba and Ruzavin, G. I. (1967)-**"The nature of mathematical knowledge"**, Thought Editorial, Moskva (Russian).
30 To see for additional details Szekely, L. (1990)-**"Motion and the dialectical view of the world"**, Studies in Soviet Thought 39, 241-255.

"From the point of view philosophical Marx was proposed penetrate to dialectic materialism in the contradictions of Infinitesimal Calculus." … "and considered that this ultimately should be settled with the implementation of dialectic method to the mathematic" …
"Marx considered the Calculus as a new degree or stage in the development of Mathematics, qualitatively superior."
"The principle of the unity of the logical and historic as a method of cognition was a key factor in the arrival in this conclusion."[31]
By other hand, research was subject to censorship. Hence, scientists and researches were denied access to some publications and research of the Western scientists, or any others deemed politically incorrect; access too many others sources was restricted. Their own research was similarly censored, some scientists were forbidden from publishing at all, many others experienced significant delays or had to agree to have their works published only in closed journals, to which access was significantly restricted. But this not only happened in the Soviet Union, in Cuba the phrase *"he is off the track ideologically"* meant a danger for the academic race.
Finally, are all considerations of Marxists are wrong? No, many works are useful and brilliants guide, but the *super valuations* of "classics" they make into platonic and absolutist, don't forget the Mathematics is a human construction, no a legacy of certain people[32].
As a mathematician, I would further say that mathematics is an awe-inspiring science, filled with mystery and wonder, and brimming with opportunities to make triumphant intellectual discoveries. It is truly one of the highest points of humankind's achievements. It offers everyone the chance to get a glimpse of the nature of the universe around us, and to learn and understand something more about the human condition. It is the most universal of our languages and the most useful of our tools; it is the most beautiful of our music and the most elegant of our poems; it is silent harmony and form; it is to some people an art.
As an undergraduate student I found the following results very beautiful:

$$1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots\frac{1}{n^2}+\ldots=\frac{\pi^2}{6}\,^{33},\quad 1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots\frac{(-1)^n}{n}+\ldots=\frac{\pi^2}{12}.$$

What do you feel about them? What about $e^{\pi i}=1$. A tale of three wonderful numbers. Arguably, that formula is the most beautiful single formula in all mathematics.

Finally, I beliefs that *mathematics is the science which deals with magnitudes (variables and constants, qualitative and quantitative), forms (abstract and concretes), patrons and rules, that it uses general methods and own techniques for study, understand and modify social, naturals and human systems and phenomena. The mathematics is a collective activity of the mathematic community, consolidated gradually in the time.*

2. What is a mathematician?

In my opinion, a mathematician is a person who not only studies mathematics but also does research in mathematics. Some mathematicians do research as well as teach Mathematics.
The ideas people entertain regarding what happens in mathematicians' heads when they are engaged in practicing their science originate no doubt from their own personal

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[31] Matute P., M.; A. Soldatov and others (1987)-**"Philosophical and methodological problems of Mathematics"**, Universidad de Oriente, Santiago de Cuba (Spanish), pp.35-36.
[32] See the introduction of Reuben Hersh in **"18 Unconventional Essays on the Nature of Mathematics"**, Springer, 2006.
[33] Cf. Capítulo 1 "A história como elemento unificador na educaçao matemática" in **"A História como un agente de cogniçao na Educaçao Matemática"**, Porto Alegre: Editora Sulina, 2006, of Fossa, J.; J. Nápoles and I. Abreu.

mathematical experiences. For no mathematicians these will mostly be confined to math classes in school or at university, where mathematics appears wearing the hat of an ancillary science. This type of experience is unfortunately prone to lead to fundamental misunderstandings that give rise to completely mistaken ideas as to what mathematics is all about: it is most emphatically not a machine-translatable aptitude for calculating according to formulae and rigid precepts that do not allow space for individual freedom. The reason for the wide prevalence of this travestied image of mathematics is arguably the fact that exams cast in this ostensibly 'objective' form are easier to implement both for preparation and assessment.

Lack of experience on the part of teachers and examiners will often lead to an aggravation of the misunderstanding[34].

By opposite mathematicians are typically interested in finding and describing *significant* which may have originally arisen from problems of calculation, but have now been abstracted to become their own problems. From much published research work of mathematicians, it may look as if the primary approach of a mathematician is to start with some given assumptions, often called axioms, and then proceed to prove other facts which follow from the assumptions according to exact rules of logic. That, however, is the finished product that gets published; it is not work in progress.

Contrary to popular belief, mathematicians are not typically any better at adding or subtracting numbers, or figuring the tip on a restaurant bill, than members of any other profession, in fact, some of the best mathematicians are notoriously bad at these tasks!
A mathematician uses numbers and symbols in many ways, from creating new theories to translating scientific and technical problems into mathematical terms. There are two types of researching mathematicians: the theoretical mathematicians, who work with pure mathematics to develop and discover new mathematical principles and theories without regard to their possible applications; and applied mathematicians, who use mathematical methods to solve practical problems in diverse areas.

To some extent, people give differing definitions of the mathematician, probably owing to the nature of their own work. We cite some examples.

- *"A mathematician is a machine for turning coffee into theorems"*, P. Erdos (1913-1996)[35].
- *"A person who can, within a year, solve* $x^2 - 92y^2 = 1$, *is a mathematician"*, Brahmagupta (598-668)[36].
- *"I have hardly ever known a mathematician who was capable of reasoning"*, Plato (429-347 b.C.[37]
- *"To be a scholar of mathematics you must be born with talent, insight, concentration, taste, luck, drive and the ability to visualize and guess"*[38].
- *"Mathematics is a dangerous profession; an appreciable proportion of us go mad, and then this particular event would be quite likely"*[39].
- *"Mathematicians are like lovers. Grant a mathematician the least principle, and he will draw from it a consequence which you must also grant him, and from this consequence another"*, B. B. Fontenelle (1657-

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34 See Reinhard Winkler-**"What is mathematics? – A subjective approach"**, http://www.dmg.tuwien.ac.at/winkler/pub/manfred-englisch.pdf
35 Rose, N. (1988)-**"Mathematical Maxims and Minims"**, Raleigh NC, Rome Press Inc.
36 Ernest, P. (1991).
37 Rose (1988).
38 Halmos, P. R. (1985)-**"I Want To Be A Mathematician"**, Washington: MAA Spectrum.
39 Littlewood, J. E. (1953)-**"A Mathematician's Miscellany"**, Methuen and Co. Ltd.

1757)[40].

• *"It is a melancholy experience for a professional mathematician to find himself writing about mathematics. The function of a mathematician is to do something, to prove new theorems, to add to mathematics, and not to talk about what he or other mathematicians have done."*[41]

In other words, mathematicians are interested not only in what happens when you adopt a particular set of rules, but also in what happens when you change the rules. For example, Lobatchevski, Bolyai, Gauss and Riemann started with Euclid's geometry, but asked *"What if parallel lines could intersect each other? How would that change things?"* And they ended up inventing an entirely new branch of geometry, which turned out to be just what Einstein needed for his theory of general relativity.

So, we can distinguish three types of mathematical activity:
1. To solve problems.
2. To demonstrate theorems or to refute conjectures[42].
3. To apply, broadly speaking, the ideas, methods, algorithms, etc. obtained in the preceding steps.

As becomes apparent from studying truly great mathematicians, the real motivations to get involved with mathematics are these activities. Only those who have some knowledge of this allure of mathematics at least from hearsay can hope to do justice to the science[43].

3. Conclusion.

Mathematics is an old, broad, and deep discipline (field of study). I think that people working to improve math education need to understand *"What is Mathematics?"* It's clear if we take in account to Philip J. Davis[44] when say *"The inter-interpretability exhibited in Mathematics Elsewhere is often cited as evidence for both unity and universality. But this depends on a definition of mathematics that is sufficiently restricted to exclude the cultural underlay. My own definition of mathematics is that it includes everything that makes its core comprehensible. (And I'm not sure how to define the core.) I see unity only in a weak sense."*

The proper stuff of mathematics is ideas and concepts. Mathematicians are called upon to describe these as accurately as possible and to ascertain whether they are categories inherent to the process of thought or whether other options are available. As opposed to empirical sciences that explore the world as it is, mathematics charts the world under the double aspects of necessity and freedom.

This is why mathematics is the least restricted and most universal science. This also accounts for its applicability too many other branches of science, this applicability, make the math a special case: *"Mathematics occupies a special position among the sciences and in the educational system. This position is determined by the fact that mathematics is an a priori science building on ideal elements abstracted from sensory experiences, and at the same time mathematics is intimately connected to the experimental sciences, traditionally not least the natural sciences and the engineering sciences. Mathematics can be decisive when formulating theories giving insight into observed phenomena, and often forms the basis for further conquests in these sciences because of its power for deduction and calculation.*

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[40] Larney, V. H. (1975)-**"Abstract Algebra: A First Course"**, Boston, Prindle, Weber and Schmidt.

[41] Hardy, G. R. (1941)-**"A Mathematician's Apology"**, London Cambridge University Press, p. 1.

[42] Thurston, W. P. (1994)-**"On proof and progress in mathematics"**, Bull. Amer. Math. Soc. 30, 161-177.

[43] An interesting point of view is Eisenberg, T. (2008)-**"Flaws and Idiosyncrasies in Mathematicians: Food for the Classroom?"** The Montana Mathematics Enthusiast, 5(1), 3-14.

[44] **"Book Review"** of SIAM News, Volume 36, Number 2, March 2003 refer to Marcia Ascher (2002)-**"Is Mathematics a Unified Whole? Mathematics Elsewhere: An Exploration of Ideas Across Cultures"**, Princeton University Press, Princeton, New Jersey, and Oxfordshire, UK.

The revolution in the natural sciences in the 1600s and the subsequent technological conquests were to an overwhelming degree based on mathematics. The unsurpassed strength of mathematics in the description of phenomena from the outside world lies in the fascinating interplay between the concrete and the abstract."[45]
Mathematics is the highest form of symbiosis between intuition and scientific precision. For it to be taught adequately requires the most holistic form of communication, i.e. communication between individuals in terms of states of mind.

Any discipline (an organized, formal field of study) such as mathematics tends to be defined by the types of problems it addresses[46], the methods it uses to address these problems, and the results it has achieved. One way to organize this set of information is to divide it into the following three categories (of course, they overlap each other):

1. Mathematics as a human endeavor[47]. For example, consider the math of measurement of time such as years, seasons, months, weeks, days, and so on. Or, consider the measurement of distance, and the different systems of distance measurement that developed throughout the world. Or, think about math in art, dance, and music. There is a rich history of human development of mathematics and mathematical uses in our modern society.
2. Mathematics as a discipline. You are familiar with lots of academic disciplines such as archeology, biology, chemistry, economics, history, psychology, sociology, and so on. Mathematics is a broad and deep discipline that is continuing to grow in breadth and depth. Nowadays, a Ph.D. research dissertation in mathematics is typically narrowly focused on definitions, theorems, and proofs related to a single problem in a narrow subfield in mathematics.
3. Mathematics as an interdisciplinary language and tool. Like reading and writing, math is an important component of learning and *doing* (using one's knowledge) in each academic discipline. Mathematics is such a useful language and tool that it is considered one of the *basics* in our formal educational system.

To a large extent, students and many of their teachers tend to define mathematics in terms of what they learn in math courses, and these courses tend to focus on instrumental view of mathematics. The instructional and assessment focus tends to be on basic skills and on solving relatively simple problems using these basic skills. As the three-component discussion given above indicates, this is only part of mathematics.

Even within the third component, it is not clear what should be emphasized in curriculum, instruction, and assessment. The issue of basic skills versus higher-order skills is particularly important in math education. How much of the math education time should be spent in helping students gain a high level of accuracy and automaticity in basic computational and procedural skills? How much time should be spent on higher-order skills such as problem posing, problem representation, solving complex problems, and transferring math knowledge and skills to problems in non-math disciplines?

I take as mathematics that which in the course of history has evolved as the product of the activity of mathematicians and has to a great extent been standardized, conventionalized and corroborated by extended experience and manifold practical usages.

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[45] Hansen, V. L. (2003)-"**Popularizing Mathematics: From Eight to Infinity"**, ICM 2002·Vol. III, ·1–3, in arXiv:math/0305019v1 [math.HO] 1 May 2003.

[46] In the Lecture **"The History of Mathematics across yours problems"** (in Spanish) held in the VIII SNHM, Belen do Pará, Brazil, April 5 to 8 of current year, we present the different types of problems resolved and how they have been changing throughout history. In a next work we will give more details. Also cf. Grinin, L. E.-"**Periodization of History: A theoretic-mathematical analysis"**, in History & Mathematics: Analyzing and Modeling Global Development, Grinin, L. E., de Munck V., Korotayev A. (eds.), pp. 10–38. Moscow: KomKniga.

[47] Manin, Yu. I. (2007)-**"Mathematical knowledge: internal, social and cultural aspects, in Mathematic and Culture"**, M. Emmer (Ed.), Ch.2, Springer, 2004, preprint arXiv:math/0703427v.

It is the concepts, methods, notations, basic assumptions, etc. which rather unanimously are considered to be mathematical that make up *mathematics*. I admit that possibly those concepts and methods might differ depending on basic views about the nature of mathematics[48].

A fallible perspective provides a powerful additional source of arguments for the social responsibility of both mathematics and its teaching. It also fits well with the emerging constructivist views of learning in mathematics and science education. But all of these benefits can be had without this philosophical commitment.
Finally, the Mathematical one should be considered as a class of mental activity, a social construction that contains conjectures, tests and refutations whose results are subjected to revolutionary changes and whose validity, therefore, it can be judged with relationship to an it pierces social and cultural, contrary to the absolutist vision (platonic) of the mathematical knowledge.
The affirmed thing takes previously us to the following open questions that will be treated in other works[49]:

- What about *applied mathematics*?[50]
- What about the *mathematical proof*?[51]
- What is the *history of mathematics*?[52]
- What is *relevant* and/o *necessary* for the mathematics education?[53]

[48] Dörfler, W. (2000)-**"Mathematics, Mathematics Education and Mathematicians: An Unbalanced Triangle"**, International Commission on Mathematical Instruction http://www.emis.de/mirror/IMU/ICMI/bulletin/49/Mathematics.html
[49] In other direction cf for example "**What is good Mathematics?"** of Terence Tao in arXiv:math/0702396v1 [math.HO] 13 Feb 2007.
[50] Maybe start with Wigner, E. P. (1960)-**"The Unreasonable Effectiveness of Mathematics in the Natural Sciences"**, Communications in Pure and Applied Mathematics 13: 1-14.
[51] Reid, D. A. (2002)-**"What is proof?"**, International Newsletter on the Teaching and Learning of Mathematical Proof, June, available online in http://www-didactique.imag.fr/preuve
[52] Many historians deal prefer with this question, see works of Heiede, Hersh, Bagni, Furinghetti, Garciadiego, Grattan Guinnes, etc.
[53] **"Mathematical Education"** in Notices of the AMS 37 (1990), 844– 850, of William P. Thurston, online in arXiv:math/0503081v1 [math.HO] 4 Mar 2005.