Complex Learning through Cognitively Demanding Tasks

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Abstract: *The world's increasing complexity, competitiveness, interconnectivity, and dependence on technology generate new challenges for nations and individuals that cannot be met by "continuing education as usual" (The National Academies, 2009). With the proliferation of complex systems have come new technologies for communication, collaboration, and conceptualization. These technologies have led to significant changes in the forms of mathematical thinking that are required beyond the classroom. This paper argues for the need to incorporate future-oriented understandings and competencies within the mathematics curriculum, through intellectually stimulating activities that draw upon multidisciplinary content and contexts. The paper also argues for greater recognition of children's learning potential, as increasingly complex learners capable of dealing with cognitively demanding tasks.*

Keywords: complex systems; complex learning; models and modeling; 21st century technologies; teaching and learning

> Although educational reformers have disagreed on many issues, there is a widely shared concern for enhancing opportunities for students to *learn mathematics with understanding* and thus a strong interest in promoting *teaching mathematics for understanding*. (Silver, Mesa, Morris, Star, & Benken, 2009, P.503).

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The Mathematics Enthusiast, **ISSN 1551-3440, Vol. 8, no.3**, pp.483-506

English

Introduction

In recent decades our global community has rapidly become a knowledge driven society, one that is increasingly dependent on the distribution and exchange of services and commodities (van Oers, 2009), and one that has become highly inventive where creativity, imagination, and innovation are key players. At the same time, the world has become governed by complex systems—financial corporations, the World Wide Web, education and health systems, traffic jams, and classrooms are just some of the complex systems we deal with on a regular basis. For all citizens, an appreciation and understanding of the world as interlocked complex systems is critical for making effective decisions about one's life as both an individual and as a community member (Bar-Yam, 2004; Jacobson & Wilensky, 2006; Lesh, 2006).

Complexity—the study of systems of interconnected components whose behavior cannot be explained solely by the properties of their parts but from the behavior that arises from their interconnectedness—is a field that has led to significant scientific methodological advances. With the proliferation of complex systems have come new technologies for communication, collaboration, and conceptualization. These technologies have led to significant changes in the forms of mathematical thinking that are needed beyond the classroom. For example, technology can ease the thinking needed in information storage, representation, retrieval, and transformation, but places increased demands on the complex thinking required for the interpretation of data and communication of results. Computational skills alone are inadequate here—the ability to interpret, describe, and explain data and communicate results of data analyses is essential (Hamilton, 2007; Lesh, 2007a; Lesh, Middleton, Caylor & Gupta, 2008).

The rapid increase in complex systems cannot be ignored in mathematics education. Indeed, educational leaders from different walks of life are emphasizing the importance of developing students' abilities to deal with complex systems for success beyond school. Such abilities include: constructing, describing, explaining, manipulating, and predicting complex systems; working on multi-phase and multi-component component projects in which planning, monitoring, and communicating are critical for success; and adapting rapidly to ever-evolving conceptual tools (or complex artifacts) and resources (Gainsburg, 2006; Lesh & Doerr, 2003; Lesh & Zawojewski, 2007).

In this article I first consider future-oriented learning and then address some of the understandings and competencies needed for success beyond the classroom, which I argue need to be incorporated within the mathematics curriculum. A discussion on complex learners and complex learning, with mathematical modeling as an example, is presented in the remaining section.

Future-oriented learning

> Every advanced industrial country knows that falling behind in science and mathematics means falling behind in commerce and property. (Brown, 2006).

English

Many nations are highlighting the need for a renaissance in the mathematical sciences as essential to the well-being of all citizens (e.g., Australian Academy of Science, 2006; Pearce, Flavell, & Dao-Cheng, 2010; The National Academies, 2009). Indeed, the first recommendation of The National Academies' *Rising above the Gathering Storm* (2007) was to vastly improve K-12 science and mathematics education. Likewise the Australian Academy of Science has indicated the need to address the "critical nature" of the mathematical sciences in schools and universities, especially given the unprecedented, worldwide demand for new mathematical solutions to complex problems. In addressing such demands, the Australian Academy emphasizes the importance of interdisciplinary research, given that the mathematical sciences underpin many areas of society including financial services, the arts, humanities, and social sciences.

The interdisciplinary nature of the mathematical sciences is further evident in the rapid changes in the nature of the problem solving and reasoning needed beyond the school years (Lesh, 2007b). Indeed, numerous researchers and employer groups have expressed concerns that schools are not giving adequate attention to the understandings and abilities that are needed for success beyond school. For example, potential employees most in demand in the mathematical sciences are those that can (a) interpret and work effectively with complex systems, (b) function efficiently and communicate meaningfully within diverse teams of specialists, (c) plan, monitor, and assess progress within complex, multi-stage projects, and (d) adapt quickly to continually developing technologies

(Lesh, 2008). Research indicates that such employees draw effectively on interdisciplinary knowledge in solving problems and communicating their findings. Furthermore, although such employees draw upon their school learning, they do so in a flexible and creative manner, often generating or reconstructing mathematical knowledge to suit the problem situation (unlike the way in which they experienced mathematics in school; Gainsburg 2006; Hamilton 2007; Zawojewski, Hjalmarson, Bowman, & Lesh, 2008). Indeed, such employees might not even recognize the relationship between their school mathematics and the mathematics they apply in solving problems in their daily work activities. We thus need to rethink the nature of the mathematical learning experiences we provide students, especially those experiences we classify as "problem solving;" we also need to recognize the increased capabilities of students in today's era.

In his preface to the book, *Foundations for the Future in Mathematics Education*, Lesh (2007b) pointed out that the kinds of mathematical understandings and competencies that are targeted in textbooks and tests tend to "represent only a shallow, narrow, and often non-central subset of those that are needed for success when the relevant ideas should be useful in 'real life" situations" (p. viii). Lesh's argument raises a number of issues, including:

> What kinds of understandings and competencies should be emphasized to reduce the gap between the mathematics addressed in the classroom (and in standardized testing), and the mathematics needed for success beyond the

classroom?

How might we address the increasing complexity of learning and learners to advance their mathematical understanding within and beyond the classroom?

Understandings and competencies for success beyond the classroom

The advent of digital technologies changes the world of work for our students. As Clayton (1999) and others (e.g., Hoyles, Noss, Kent, & Bakker, 2010; Jenkins, Clinton, Purushotma, Robinson & Weigel, 2006; Lombardi & Lombardi, 2007; Roschelle, Kaput, & Stroup, 2000) have stressed, the availability of increasingly sophisticated technology has led to changes in the way mathematics is being used in work place settings; these technological changes have led to both the addition of new mathematical competencies and the elimination of existing mathematical skills that were once part of the worker's toolkit.

Studies of the nature and role of mathematics used in the workplace and other everyday settings (e.g., nursing, engineering, grocery shopping, dieting, architecture, fish hatcheries) are important in helping us identify some of the key understandings and competencies for the 21st century (e.g., de Abreu, 2008; Gainsburg, 2006; Hoyles et al., 2010; Roth, 2005). A major finding of the 2002 report on workplace mathematics by Hoyles, Wolf, Molyneux-Hodgson and Kent was that basic numeracy is being displaced as the minimum required mathematical competence by an ability to apply a much wider range of mathematical concepts in using technological tools as part of working practice. Although we cannot simply list a number of mathematical competencies and assume these can be automatically applied to the workplace setting, there are several that employers generally consider to be essential to productive outcomes (e.g., Doerr & English, 2003; English, 2008; Gainsburg, 2006; Lesh & Zawojewski, 2007). In particular, the following are some of the core competencies that have been identified as key elements of productive and innovative work place practices (English, Jones, Bartolini Bussi, Lesh, Tirosh, & Sriraman, 2008; Hoyles et al., 2010). I believe these competencies need to be embedded within our mathematics curricula:

- Problem solving, including working collaboratively on complex problems where planning, overseeing, moderating, and communicating are essential elements for success;
- Applying numerical and algebraic reasoning in an efficient, flexible, and creative manner;
- Generating, analyzing, operating on, and transforming complex data sets;
- Applying an understanding of core ideas from ratio and proportion, probability, rate, change, accumulation, continuity, and limit;
- Constructing, describing, explaining, manipulating, and predicting complex systems;
- Thinking critically and being able to make sound judgments, including being able to distinguish reliable from unreliable information sources;

- Synthesizing, where an extended argument is followed across multiple modalities;
- Engaging in research activity involving the investigation, discovery, and dissemination of pertinent information in a credible manner;
- Flexibility in working across disciplines to generate innovative and effective solutions.
- Techno-mathematical literacy (a "techno-mathematical literacy, where the mathematics is expressed through technological artefacts." Hoyles et al., 2010, p. 14).

Although a good deal of research has been conducted on the relationship between the learning and application of mathematics in and out of the classroom (e.g., de Abreu 2008; Nunes & Bryant 1996; Saxe 1991), we still know comparatively little about students' mathematical capabilities, especially problem solving, beyond the classroom. We need further knowledge on why students have difficulties in applying the mathematical concepts and abilities (that they presumably have learned in school) outside of school—or in classes in other disciplines.

A prevailing explanation for these difficulties is the context-specific nature of learning and problem solving, that is, competencies that are learned in one situation take on features of that situation; transferring them to a new problem situation in a new context poses challenges (Lobato 2003). This suggests we need to reassess the nature of the typical mathematical problem-solving experiences we give our students, with respect to the nature of the content and how it is presented, the problem contexts and the extent of their real-world links, the reasoning processes likely to be fostered, and the problem-solving tools that are available to the learner (English & Sriraman, 2010). This reassessment is especially needed, given that "problems themselves change as rapidly as the professions and social structures in which they are embedded change" (Hamilton, 2007, p. 2). The nature of learners and learning changes likewise. With the increasing availability of technology and exposure to a range of complex systems, children are different types of learners today, with a potential for learning that cannot be underestimated.

Complex learners, complex learning

Winn (2006) warned of the "dangers of simplification" when researching the complexity of learning, noting that learning is naturally confronted by three forms of complexity—the complexity of the learner, the complexity of the learning material, and the complexity of the learning environment (p. 237). We cannot underestimate these complexities. In particular, we need to give greater recognition to the complex learning that children are capable of—they have greater learning potential than they are often given credit for by their teachers and families (English, 2004; Lee & Ginsburg, 2007; Perry & Dockett, 2008; *Curious Minds*, 2008). They have access to a range of powerful ideas and processes and can use these effectively to solve many of the mathematical problems they meet in daily life. Yet their mathematical curiosity and talent appear to wane as they progress through school, with current educational practice missing the goal of cultivating students' capacities (National Research Council, 2005; *Curious Minds*, 2008). The words of Johan van Benthem and Robert Dijkgraaf, the initiators of *Curious Minds* (2008), are worth quoting here:

What people say about children is: "They can't do this yet."

We turn it around and say: "Look, they can already do this."

And maybe it should be: "They can still do this now."

As Perry and Dockett (2008) noted, one of our main challenges here is to find ways to utilize the powerful mathematical competencies developed in the early years as a springboard for further mathematical power as students progress through the grade levels. I offer three interrelated suggestions for addressing this challenge:

1. Recognize that learning is based within contexts and environments that we, as educators shape, rather than within children's maturation (Lehrer & Schauble, 2007).
2. Promote active processing rather than just static knowledge (*Curious Minds*, 2008).
3. Create learning activities that are of a high cognitive demand (Silver et al., 2009).

In the remainder of this paper I give brief consideration to these suggestions. In doing so, I argue for fostering complex learning through activities that encourage knowledge generation and active processing. While complex learning can take many forms and involve numerous factors, there are four features that I consider especially important in advancing students' mathematical learning. These appear in figure 1.

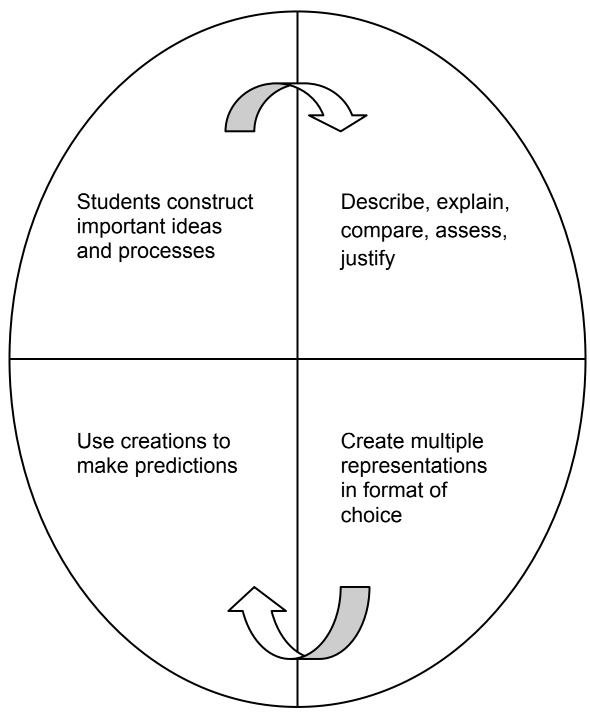

Figure 1. Key Features of Complex Learning

Research in the elementary and middle school indicates that, with carefully designed and implemented learning experiences, we can capitalize on children's conceptual resources and bootstrap them towards advanced forms of reasoning not typically observed in the regular classroom (e.g., English & Watters, 2005; Ginsburg, Cannon, Eisenband, & Pappas, 2006; Lehrer & Schauble, 2007). Most research on young students' mathematical learning has been restricted to an analysis of their actual developmental level, which has failed to illuminate their potential for learning under stimulating conditions that challenge their thinking—"Research on children's current knowledge is not sufficient" (Ginsburg et al., 2006, p.224). We need to redress this situation by exploring effective ways of fashioning learning environments and experiences that challenge and advance students' mathematical reasoning and optimize their mathematical understanding.

Recent research has argued for students to be exposed to learning situations in which they are *not* given all of the required mathematical tools, but rather, are required to create their own versions of the tools as they determine what is needed (e.g., English & Sriraman, 2010; Hamilton, 2007; Lesh, Hamilton, & Kaput, 2007). For example, long-standing perspectives on classroom problem solving have treated it as an isolated topic, with problem-solving abilities assumed to develop through the initial learning of basic concepts and procedures that are then practised in solving word ("story") problems. In solving such word problems, students generally engage in a one- or two-step process of mapping problem information onto arithmetic quantities and operations. These traditional word problems restrict problem-solving contexts to those that often artificially house and highlight the relevant concept (Hamilton, 2007). These problems thus preclude students from creating their own mathematical constructs. More opportunities are needed for students to generate important concepts and processes in their own mathematical learning as they solve thought-provoking, authentic problems. Unfortunately, such opportunities appear scarce in many classrooms, despite repeated calls over the years for engaging students in tasks that promote high-level mathematical thinking and reasoning (e.g., Henningsen & Stein, 1997; Silver et al., 2009; Stein & Lane, 1996).

Silver et al.'s recent research (2009) analyzing portfolios of "showcase" mathematics lessons submitted by teachers seeking certification of highly accomplished teaching, showed that activities were not consistently intellectually challenging across topics. About half of the teachers in the sample (N=32) failed to include a single activity that was cognitively demanding, such as those that call for reasoning about ideas, linking ideas, solving complex problems, and explaining and justifying solutions. Furthermore, the teachers were more likely to use cognitively demanding tasks for assessment purposes than for teaching to develop student understanding. While Silver et al.'s research revealed positive features of the teachers' lessons, it also indicated that the use of cognitively demanding tasks in promoting mathematical understanding needs systematic attention.

Modeling Activities

One approach to promoting complex learning through intellectually challenging tasks is mathematical modeling. Mathematical models and modeling have been interpreted variously in the literature (e.g., Romberg, Carpenter, & Kwako, 2005; Gravemeijer, Cobb, Bowers, & Whitenack, 2000; English & Sriraman, 2010; Greer, 1997; Lesh & Doerr, 2003). It is beyond the scope of this paper to address these various interpretations, however, but the perspective of Lesh and Doerr (e.g., Doerr & English, 2003; Lesh & Doerr, 2003) is frequently adopted, that is, models are "systems of elements, operations, relationships, and rules that can be used to describe, explain, or predict the behavior of some other familiar system" (Doerr & English, 2003, p.112). From this perspective, modeling problems are realistically complex situations where the problem solver engages in mathematical thinking beyond the usual school experience and where the products to be generated often include complex artifacts or conceptual tools that are needed for some purpose, or to accomplish some goal (Lesh & Zawojewski, 2007).

In one such activity, the *Water Shortage Problem*, two classes of 11-year-old students in Cyprus were presented with an interdisciplinary modeling activity that was set within an engineering context (English & Mousoulides, in press). In the *Water Shortage Problem*, constructed according to a number of design principles, students are given background information on the water shortage in Cyprus and are sent a letter from a client, the Ministry of Transportation, who needs a means of (model for) selecting a country that can supply Cyprus with water during the coming summer period. The letter asks students to develop such a model using the data given, as well as the Web. The quantitative and qualitative data provided for each country include water supply per week, water price, tanker capacity, and ports' facilities. Students can also obtain data from the Web about distance between countries, major ports in each country, and tanker oil consumption. After students have developed their model, they write a letter to the client detailing how their model selects the best country for supplying water. An extension of this problem gives students the opportunity to review their model and apply it to an expanded set of data. That is, students receive a second letter from the client including data for two more countries and are asked to test their model on the expanded data and improve their model, if needed.

Modeling problems of this nature provide students with opportunities to repeatedly express, test, and refine or revise their current ways of thinking as they endeavor to create a structurally significant product—structural in the sense of generating powerful mathematical (and scientific) constructs. The problems are designed so that multiple solutions of varying mathematical and scientific sophistication are possible and students with a range of personal experiences and knowledge can participate. The products students create are documented, shareable, reusable, and modifiable models that provide teachers with a window into their students' conceptual understanding. Furthermore, these modeling problems build communication (oral and written) and teamwork skills, both of which are essential to success beyond the classroom.

Concluding Points

The world's increasing complexity, competitiveness, interconnectivity, and dependence on technology generate new challenges for nations and individuals that cannot be met by "continuing education as usual" (The National Academies, 2009). In this paper I have emphasized the need to incorporate future-oriented understandings and competencies within the mathematics curriculum, through intellectually stimulating activities that draw upon multidisciplinary content and contexts. I have also argued for greater recognition of children's learning capabilities, as increasingly complex learners able to deal with cognitively demanding tasks.

The need for more intellectually stimulating and challenging activities within the mathematics curriculum has also been highlighted. It is worth citing the words of Greer and Mukhopadhyay (2003) here, who commented that "the most salient features of most documents that lay out a K-12 program for mathematics education is that they make an intellectually exciting program boring," a feature they refer to as "intellectual child abuse" (p. 4). Clearly, we need to make the mathematical experiences we include for our students more challenging, authentic, and meaningful. Developing students' abilities to work creatively with and generate mathematical knowledge, as distinct from working creatively on tasks that provide the required knowledge (Bereiter & Scardamalia, 2006) is especially important in preparing our students for success in a knowledge-based economy. Furthermore, establishing collaborative, knowledge-building communities in the mathematics classroom is a significant and challenging goal for the advancement of students' mathematical learning (Scardamalia, 2002).

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