The education of mathematically gifted students: Some complexities and questions

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Abstract: *In this paper I analyze some complexities in the education of mathematically gifted students. The list of issues presented in this paper is not inclusive; however, all of them seem to be typical on the international scope. Among these issues are: (1) the gap between research in mathematics education and the research in gifted education; (2) the role of creativity in the education of the gifted and the theoretical perspective on the relationship between creativity and giftedness, and (3) teaching the gifted and the teachers of gifted, including relationships between the equity principle in mathematics education and views on the education of gifted. In the paper I outline some actual research questions in the field of education of mathematically gifted.*

Key words: Educating the gifted, Mathematical creativity and giftedness, Research and practice

INTRODUCTION

Mathematics educators and researchers in mathematics education agree that any decisions made with respect to the education of mathematically talented children and adolescents should be based on research findings and on the deep understanding of mathematical thinking and learning. Following Schoenfeld (2000, 2002), who shed light on the two main purposes of research in mathematics education, I maintain that research in the field of mathematical giftedness and creativity must be carried out in two interrelated directions:

- First (theoretical) is to understand the nature of mathematical giftedness and mathematical creativity from the perspectives of thinking, teaching, and learning
- Second (applied) is to use such understanding to improve mathematics instruction in a way that helps realize mathematical giftedness and encourage mathematical creativity.

I demonstrate the shortage of systematic research in the education of mathematically gifted students, and outline some complexities in the education of gifted that require systematic research. I present some research questions that can be seen as a research agenda in the field of teaching mathematically gifted students.

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1. RESEARCH IN MATHEMATICS EDUCATION AND RESEARCH IN GIFTED EDUCATION[1]

Educational literature related to the issues of high mathematical ability, mathematical talents, mathematical giftedness and mathematical creativity contains a variety of descriptive reports, instructional guidelines, and reference materials, but research reports in the field are less common. Analysis of the research literature in the fields of gifted education and mathematics education leads to the conclusion that the studies in these two fields moved in two tangential rather than intersecting directions. The following evidence clearly illustrates that mathematics education is underrepresented in the field of gifted education and, vice versa, the research on giftedness and gifted education is underrepresented in the field of mathematics education.

1.1 Publications in Research Journals Devoted to Giftedness

During the past decade seven key journals in the field of giftedness and intelligence (Gifted Child Quarterly, High Ability Studies, Journal for the Education of the Gifted, The Journal of Secondary Gifted Education, Creativity Research Journal, and the Journal for the Education of the Gifted) published only a few articles devoted directly to mathematical giftedness or creativity. The following twelve papers, from among more than 1,000 published in the past ten years, form an almost complete list: Chamberlin & Moon, 2005; Hodge & Kemp, 2006; Hong & Aqui, 2004; Koichu & Berman, 2005; Kwon, Park & Park, 2006; Mann, 2006; Nokelainen, Tirry & Merenti-Valimaki, 2007; Olszewski-Kubilius & Lee, 2004; Reed, 2004; Sriraman, 2003; 2005; Yim, Chong, Song & Kwon, 2008). Eight of the twelve studies are clearly connected with research in mathematics education.

Mann (2006) and Sriraman (2005) perform a theoretical analysis of the relationship between mathematical creativity and mathematical giftedness. Koichu & Berman (2005), Sriraman (2003) and Yim, et al. (2008) analyze problem-solving strategies used by mathematically gifted students. Chamberlin and Moon (2005) and Kwon, Park and Park (2006) suggest developing mathematical creativity based on earlier advances in mathematics education. Reed (2004) suggests and tests approaches to teaching the gifted at geometry lessons in heterogeneous classroom. Other studies consider good performance in mathematics as one of the several characteristics of general giftedness (Hodge & Kemp, 2006; Hong & Aqui, 2004), one of

[1] If I overlooked any important publications in the course of the research, I apologize for the omissions.

the outcomes of attribution styles (Nokelainen, Tirry & Merenti-Valimaki, 2007) investigate the influence of attribution styles on the development of mathematical talent, and one of the subjects in extracurricular activities in and out of school (Olszewski-Kubilius & Lee, 2004).

1.2 Publications in Research Journals in Mathematics Education

A search of seven leading research journals in mathematics education (Journal for Research in Mathematics Education, Educational Studies in Mathematics, Journal of Mathematical Behavior, Focus on the Learning Problems in Mathematics, The International Journal on Mathematics Education – ZDM, Mathematical Thinking and Learning, and For the Learning of Mathematics) reveals that in the past decade only few publications were explicitly devoted to mathematical giftedness and creativity in these journals.

Only one publication in these journals is explicitly devoted to learning process of mathematically talented students, namely, Amit and Neria (2008) explore problem-solving strategies of talented pre-algebra students. About 10 publications in these journals directly address mathematical creativity: Presmeg (2003) and Ernest (2006) analyze and emphasize the importance of creativity in the development of mathematical meaning, and Lithner (2008) suggests a framework for analysis of mathematical activity and describes creative thinking in mathematics as opposed to imitative thinking. Liljedahl and Sriraman (2006) conduct a discussion about the meaning of mathematical creativity and its role in activities of professional mathematicians vs. mathematical activities of school children. This work provides a theoretical view on mathematical creativity, with connections to works by Polya, Hadamard, and Poincaré (for details about this theoretical perspective, see Liljedahl, 2009).

Sriraman (2009) argues that mathematical creativity is the main mechanism of the growth of mathematics as science. However he finds that the creativity "is a relatively unexplored area in mathematics and mathematics education." (p. 13). In his paper Sriraman provides a critical analysis of characteristics of mathematical creativity from different theoretical perspectives. Plucker and Zabelina (2009) stress the importance of defining creativity, admit the lack of literature that deals with the concept of creativity in mathematical education and provide their own definition of general creativity. Based on this definition they discuss domain-specific and domain-general creativity. Hoyles (2001) analyzes the role that a computer-based learning environment can play in navigation between skills and creativity in teaching mathematics. This analysis leads to observation that technology-based inquiry opens opportunities for the advancement of students' mathematical creativity. Bibby (2002) provides a view from an elementary school mathematics classroom on the opportunities of simultaneous development of students' logical and creative reasoning. Shriki (2009) analyses pre-service mathematics teachers' views on mathematical creativity and demonstrates that their knowledge associated with mathematical creativity is insufficient for the discussion of the subject. Huckstep and Rowland (2000) review *Creative Mathematics*, a book by Upitis, Phillips & Higginson (1997), which provides insightful ideas for creative mathematical activities in school.

Note that numerous publications in Mathematics Education journals, in the past ten years, use the words “creative”, “inventive,” and “original” in their descriptions of mathematical activities suggested to students and of students’ mathematical performance. Mathematics educators and researchers design, describe, and explore mathematical activities with a high potential for the development of mathematical creativity in school children. Works devoted to “doing mathematics” in classroom, to inquiry based classrooms and students' autonomy in such classrooms, to active construction of mathematical knowledge, and to students heuristics in problem solvingare implicitly related to mathematical creativity among students. However, in these works the words “creativity” and “inventiveness” are not part of terminology in the analysis of students’ mathematical reasoning and the teachers’ role in the classroom. Mathematics education must therefore pay more attention to research of different kinds of mathematical activities, with a clear focus on students’ creative thinking and giftedness.

1.3 Other sources

A small number of publications in other journals focused on specific issues in the mathematical reasoning and problem solving of the gifted population. Among them are Gorodetsky & Klavir (2003); Livne, Livne & Milgram (1999); and Chiu (2009), who examine students' creativity in mathematical problem solving and suggest ways for analyzing students' creativity.

Several other research publications about students and adults with high mathematical abilities can be found in the Journal of Educational Psychology, Psychological Science Journal, and Journal of Applied Psychology. These studies, mostly by Lubinski, Benbow and their colleagues, are a part of larger longitudinal study that was precipitated by the study of Mathematically Precocious Youth (SMPY) at John Hopkins University which was initially spearheaded by Julian Stanley in earlier 1980s. For example, Lubinski, Webb, Morelock and Benbow (2001), on the basis of 10 years of observations, demonstrated that early identified distinctions in intellectual strength predicted differences in the developmental trajectories and occupation pursuits of highly talented individuals. They also demonstrated the effectiveness of acceleration for individual cases in their 20-year follow-up study on 1975 mathematically gifted adolescents (top 1%). They demonstrate that earlier identified gender differences in mathematical reasoning of the participants predicted differential education and occupational outcome all of which were successful. Other publications by Lunibski and Benbow explore innovative evaluation tools for the identification of mathematical talents. For example, Lubinski & Benbow (2000) demonstrate that combination of theory of work adjustment concepts and psychometric methods facilitate positive development of talented youths. Another study (Webb, Lubinski & Benbow, 2007) demonstrates that spatial ability is significant for talent identification. Still, these studies focus mainly on general psychological characteristics of individuals and do not explore learning and thinking processes in mathematically gifted school students as associated with the contemporary theories of Mathematics Education (see elaboration and examples in Leikin, 2009a).

Lately there were several edited volumes devoted to these issues. Sriraman's (2008) monograph *Creativity, Giftedness, and Talent Development in Mathematics* includes contributions devoted to creativity and giftedness in mathematics, offers new perspectives for talent development in mathematics classroom and gives insights into the psychology of creativity and giftedness. However, the editor stressed the lack of systematic research of talent development in mathematics education. Leikin, Berman and Koichu (2009) edited a volume entitled *Creativity in Mathematics and the Education of Gifted Students*. As a result of a consolidated effort of a group of experts in the fields of mathematics education, psychology, educational research, mathematics and policy making the book puts in the foreground mathematical creativity and mathematical giftedness as important topics in educational research. The book includes several reports on the empirical studies related to mathematical creativity and giftedness along with theoretical framework for the analysis of mathematical creativity and giftedness. The editors stress the importance of empirical research in the field that must be performed in various spheres related to the education and identification of mathematically able students (see Leikin, 2009a).

1.4 International forums

At the international level one can see raising awareness of the importance of gifted education in mathematics. This awareness is reflected in a number of international forums that lately have focused their work on mathematical creativity and giftedness. ICME conferences twice included Topic Study Group (TSG) "Activities and programs for gifted students" (TSG-4 at ICME-10 in 2004 http://www.icme-organisers.dk/tsg04/; TSG-6 at ICME-11 in 2008 http://tsg.icme11.org/tsg/show/7). At ICME 11 Discussion group "Promoting creativity for all students in mathematics education"took place along with TSG-6 mentioned above (DG-9, http://dg.icme11.org/tsg/show/10). In summer 2008 ICMI Study-16 "Mathematical challenges in and beyond the classroom" discussed a variety of issues related to education of mathematically talented students. The results of the elaborated discussion by all the participants are expressed in the corresponding ICMI Study Volume (Barbeau & Taylor, 2009).

Since 1999 the main forum (founded by Meissner and Sheffield) that unites educational researchers, mathematicians and mathematics educators interested in education of mathematically gifted and development of mathematical creativity has been International Conference on Creativity in Mathematics and the Education of Gifted Students. Each of the 5 conferences (1999 – in Muenster, Germany; 2002—in Riga, Latvia; 2003—in Rousse, Bulgaria; 2006 – in Budejovice, Czech Republic; 2008 – in Haifa, Israel) issued proceedings including the conference papers. Eventually in Riga, Latvia in summer 2010 the participants of the conference established the International Group for Mathematical Creativity and Giftedness (MCG) (for the information about the group and the conferences see http://igmcg.org)

To sum it up, the discussion presented in this section of the paper underscores the need for advancement of the research-based perspectives on mathematical talent and mathematical creativity both in the direction of characterization of individuals with high mathematical ability (both analytical and creative) and the development of high mathematical abilities. Since Krutetskii's (1976) fundamental research on characterization of mathematical abilities in gifted students, there were performed several studies focusing very specifically on issues related to mathematics reasoning and problem solving of gifted students. Using the criteria suggested by Schoenfeld (2000, 2002) for theories and models in mathematics education, I argue that most of the existing works in the field must be further examined with respect to their explanatory and predictive power, scope, and replicability. The following sections in this paper describe several complexities in the education of mathematically gifted student that can become focal points of the systematic research in the field of mathematics education.

2. MATHEMATICAL CREATIVITY AND MATHEMATICAL GIFTEDNESS

One of the research questions that requires special attention of the mathematics education community is the relationship between mathematical creativity and mathematical giftedness.

2.1 Creativity as property of professional mathematicians vs. creativity for all

One of the complexities related to the relationship between mathematical giftedness and mathematical creativity is rooted in the contrast between viewing mathematical creativity as a property of the mind of the professional mathematicians (Subotnik, Pillmeier & Jarvin, 2009; Sriraman, 2005; Liljedahl & Sriraman, 2006) and the opinion that mathematical creativity must and can be developed in all students (Sheffield, 2009; Yerushalmy, 2009; Hershkoivits, Peled & Littler, 2009).

According to Subotnik et al. (2009) creativity is fundamental to the work of a professional mathematician. In the course of their work, mathematicians find and solve problems that are substantive and challenging. Subotnik et al. (2009) describe the development of ability into competence, expertise, and finally scholarly productivity/artistry and argues that mathematicians need an array of psychosocial skills to be successful in such a highly competitive intellectual arena. Similarly Ervynck (1991) considers mathematical creativity as one of the characteristics of advanced mathematical thinking. Ervynck connected mathematical creativity with advanced mathematical thinking and considered it as the ability to formulate mathematical objectives and find inherent relationships among them.

Sriraman in his conversation with Liljedahl on the notion of mathematical creativity (Liljedahl & Sriraman, 2006) suggests that at the professional level mathematical creativity can be defined as "the ability to produce original work that significantly extends the body of knowledge (which could also include significant syntheses and extensions of known ideas)" or "opens up avenues of new questions for other mathematicians" (ibid. p. 18). Sriraman (2005) considers mathematical creativity as one of the characteristics of advanced research mathematicians. He defined seven levels of mathematical ability associated with mathematical creativity and giftedness. The abilities of professional mathematicians, according to this model, are at levels 5, 6, and 7, and he differentiated these levels with respect to the mathematicians' measure of creativity: "Level 5" mathematicians are productive in mathematical research and have high levels of analytic and practical abilities, whereas creative mathematicians (levels 6 and 7) have higher levels of synthetic abilities, which allow them to "open up new research vistas for other mathematicians" (*ibid*., p. 30).

Sriraman (2005) stresses that creativity in school mathematics obviously differs from the creativity of professional mathematicians: "At the K–12 level, one normally does not expect works of extraordinary creativity; however, it is certainly feasible for students to offer new insights". Furthermore Silver (1997) and Sheffield (2009) address "creativity to all students" and consider solving problems and problem posing as main tools for the development of mathematical creativity in all the students. Along with this position Liljedahl and Sriraman (2006) argue that at school levels or even the undergraduate level "it is feasible for students to offer new insights/solutions" in mathematics. These insights/solutions are usually new with respect to mathematics the students have already learned and the problems they have already solved. Taking a developmental point of view, Sheffield (2009) suggests a continuum of mathematical proficiency through the development of creative ability in mathematics: innumeraters → doers → computers → consumers → problem solvers → problem posers → creators.

Viewing personal creativity as a characteristic that can be developed in schoolchildren requires distinction between *relative* and *absolute* creativity (Leikin, 2009). *Absolute creativity* is associated with "great historical works" (in terms of Vygotsky, 1930/1984), with discoveries at a global level. For example, examples of absolute creativity may be seen in discoveries of Fermat, Hilbert, Riemann and other prominent mathematicians (Sriraman, 2005). *Relative creativity* refers to discoveries of a specific person in a specific reference group. This type of creativity refers to the human imagination as it creates anything new (Vygotsky, 1930/1984).

2.2 The relationship between mathematical giftedness and mathematical creativity

While connecting between high mathematical abilities and mathematical creativity researchers express a diversity of views. Some researchers claim that creativity is a specific type of giftedness (e.g., Sternberg, 1999, 2005), others feel that creativity is an essential component of giftedness (Renzulli, 1978, 1986), while other researchers suggest that these are two independent characteristics of human beings (Milgram & Hong, 2009). Thus analysis of the relationships between creativity and giftedness with specific focus on the fields of mathematics is important for better understanding of the nature of mathematical giftedness.

Creative thinking includes finding different solutions and interpretations, making various mathematical connections, applying different techniques, and thinking originally and unusually. In this sense creativity is a part of the problem solving process and one of the outcomes of learning mathematics. Another (overlooked) perspective on creativity we find in works of Vygotsky who stresses the role of creativity in the process of knowledge development, abstraction and generalization. Vygotsky (1930/1984) argued that creativity (imagination) is one of the basic mechanisms that allow development of new knowledge. A child activates imagination when connecting new and previously known concepts, when elaborating the known constructs, and when developing abstract notions. Thus imagination (or creativity) is a basic component of knowledge construction. Thus we deduce as follows about the complexity in the relationship between creativity and knowledge development: *to have knowledge is a necessary condition for a person to be creative while to have imagination is a necessary condition for knowledge construction.* These relationships are one of the central issues for investigation by mathematics education researchers.

Providing a precise and broadly accepted definition of mathematical creativity is an extremely difficult and probably unachievable task (Haylock, 1987; Liljedahl & Sriraman, 2006; Mann, 2006; Sriraman, 2005). Mann (2006) affirmed that analysis of the research attempting to define mathematical creativity revealed how the lack of an accepted definition for mathematical creativity hinders research effort. Following these observations, Leikin (2009a) suggested a model for the evaluation of creativity using Multiple Solution Tasks. The model includes operational definitions and a corresponding scoring scheme for the evaluation of creativity, which is based on three components: fluency, flexibility, and originality -- as suggested by Torrance (1974). For the evaluation of originality it utilizes Ervynk's (1991) insight-related levels of creativity in combination with conventionality of the solutions which comprises students' educational history in mathematics.

In several recent studies, that accepted developmental perspective on mathematical creativity, I and my colleagues implement the model for evaluation of mathematical creativity through Multiple Solution Tasks (Leikin, 2009b). In two of the studies (Levav-Waynberg & Leikin, 2009 and Guberman & Leikin, in preparation; Leikin, Levav-Waynberg & Guberman, accepted) we examine development of mathematical creativity through mathematical instructions. Among other findings, we discovered that as the result of systematic implementation of Multiple Solution Tasks in mathematical instruction, students' flexibility and fluency significantly increased. Students' originality, however, decreased non-significantly, and resulting in a non-significant decrease in the creativity. Findings related to flexibility and fluency are naturally desirable.

Results related to originality have a reasonable explanation: when the students' flexibility increases, more students in the group produce more solutions and it becomes more difficult to produce a unique solution. Following these findings, we question the possibility of developing originality and hypothesize that in the fluency-flexibility-originality triad, fluency and flexibility are of a dynamic nature, whereas originality is a "gift".

Finally, our findings demonstrate that originality appeared to be the strongest component in determining creativity and the strength of the relationship between creativity and originality can be considered as validating our model, being consistent with the view of creativity as invention of new products or procedures. At the same time, our studies demonstrate that this view is true for both the absolute and the relative levels of creativity. We also assume that one of the ways of identification mathematically gifted students is by means of originality of their ideas and solutions.

Based on the above observations it is clear that systematic research should be performed to examine different ways of promotion of mathematical creativity in school students, identification of creative talents in school students, and between understanding of the relationship between mathematical creativity and mathematical giftedness.

3 TEACHING THE GIFTED AND TEACHERS OF THE GIFTED

3.1 Approaches and frameworks for teaching the gifted

Subbotnik et al. (2009) stressed that during the past 25 years multiple educational programs for talented youths have been proposed. Examples include Parnes's creative problem solving method (Parnes, Noller & Biondi, 1977), Renzulli's enrichment triad model (Renzulli, 1978; Renzulli & Reis, 1985), Johns Hopkins University acceleration program (Fox, 1974; Stanley, 1991), Tannenbaum's (1983) enrichment matrix, and many others. According to Nevo and Rachmel

(2009) programs for gifted education can be ranked by the intensity of the program, the most intensive being found in special schools for mathematically gifted students (Vogeli, 1997).

Usually characteristics of the effective learning environments for mathematically talented students follow specific characteristics of this population. These students tend to use self-regulatory learning strategies more often and more effectively than other students, and are better able to transfer them to novel tasks. In their review of research on the thinking process of highly able children, Shore and Kanevsky (1993) argued that if the gifted think more quickly and make fewer errors, and then we need to teach more quickly. Shore and Kanevsky stress that this is not entirely the case; adjustments have to be made in methods of learning and teaching, to take into account individual thinking differences Nisbet (1990) suggested several approaches to promote self-regulation in learning in science teaching that seem to be applicable to mathematics education:

- Talking aloud. According to this approach the teacher talks aloud while solving a problem so that the pupils can visualize work-out.
- Cognitive apprenticeship. This approach requires the teacher to demonstrate to students the processes that experts use to handle complex tasks, guiding the pupil via experiences.
- Discussion involves analysis of the processes of argument.
- Cooperative learning, which requires that pupils explain their reasoning to each other.
- Socratic questioning is based on careful questioning to force pupils to explain their thought processes and their arguments.

Nevo (2004) distinguished the methods of nurturing gifted children that exist around the world, and classified them according to three basic approaches relating to the capabilities of gifted students:

- *Acceleration* is usually defined as learning topics within the areas of studentsat accelerated pace. This can be expressed in early entrance into school, skipping grades, Advanced Placement, and/or earlier entrance to the university courses (Southern & Jones, 1991; Van Tassel-Baska, 2004a, b).
- *Broadening* is considered as studying a additional topics and subjects simultaneously with usual school mathematics. For example, studying extra-curricula topics in mathematical circles relates to the broadening approach. (e.g., Fomin, Genkin & Itenberg, 2000), learning belong to this approach. .

- *Deepening* is usually associated with studying curricular topic at greater depth than prescribed by the curriculum or school textbooks. Deepening can include, for example, learning underlying rules for regular curricular topics.

Some of these approaches are highly appropriate for in-school framework as special classes for students with high abilities in mathematics can differ in the manner in which ability grouping is managed: through subject-based streaming, the provision of special classes, or the availability of special schools. Other activities such as math clubs, competitions, and student conferences can be found both in school and out of school. The integration of students in university courses, virtual courses, and personal mentoring are typical out-of-school solutions (Leikin, 2009a).

Despite the variety of frameworks for the education ofmathematically gifted students, there is lack of empirical data about this field. It is necessary to conduct systematic empirical studies on various programs to gain better understanding of their effectiveness and suitability for the realization of the students' mathematical potential and the development of their creativity. We lack theoretical characterizations of effective courses and programs for mathematically talented students. Research should be directed at the theoretical characterizations of programs for students with high mathematical abilities

3.2 Equity principle and ability grouping

Some educational communities have provided special ability-grouping-based frameworks for treating mathematically gifted students. Among them special schools, as, for example, Kolmogorov's Schools in Russia (Kolmogorov, 1965; Kolmogorov, Vavilov & Tropin, 1981), or centers for gifted and talented youth, as, for example, CTY at John Hopkins University (http://cty.jhu.edu/about/index.html). These schools have shown to be effective and exciting frameworks for the education of gifted students (e.g. Karp, 2009; Vogeli, 1997). Nevertheless, some opponents of ability grouping argue that it contradicts the equity principle in mathematics education pronounced by the National Council of Teachers of Mathematics (NCTM, 1989). According to this principle "all students, regardless of their personal characteristics, backgrounds, or physical challenges must have opportunities to study – and the support to learn – mathematics". At the same time, special schools and classes for gifted may be seen as the expression of the equity principle because education must provide equal opportunities to all students to learn, realize their potential, which is comprised of intellectual abilities, personality and affective characteristics (NCTM, 1995; Sheffield, 1999; Leikin, 2009a). The central function of the educational system is providing each and every student regardless of his/her social and economical status with learning opportunities that match their potential and promote it to the maximal extent.

Thus interpretation of the equity principle as associated with the education of mathematically gifted students is not trivial. In late 80th – earlier 90th the equity principle was (mis)interpreted as a recommendation to provide all students with identical instruction. The drive for social justice and the democratic view of education led to the cancellation of ability tracking in mathematics, and domination of heterogeneous mathematics education. Very often at a local level, school principles, mathematics coordinators or mathematics teachers echo this policy and held a mid-ability oriented position based on reasonable argument: If I will let high achievers learn "alone" then the average students will have nowhere to grow.

This conception also received a research base when in late 80s heterogeneous classroom was shown as an effective learning environment especially for students with middle level of abilities. Cahan, Linchevski and Igra (1995), Cahan and Linchevski (1996) and Linchevski and Kutscher (1998) demonstrated that mixed-ability grouping is more beneficial for mid-level student that grouping with low achieving students and that high achievers do not differ in their learning outcomes as either kind of ability grouping. The debate on the necessity of ability grouping is legitimate, and both proponents and opponents of heterogeneous mathematics education use valid arguments to justify their positions. NCTM (2000) re-conceptualized the equity principle and stressed that "Equity does not mean that every student should receive identical instructions; instead it demands that reasonable and appropriate accommodations be made to promote access and attainment for all students" (ibid., p. 12).

Ability grouping was shown as one of the ways of achieving the equity principle in the education of mathematically gifted students. Ability grouping may be essential for education of gifted both from cognitive and affective perspectives (Davis & Rimm, 2002), and it ought to supply special education to mathematically gifted students and prevent talent loss (Milgram & Hong, 2009). On the other hand ability grouping is still questionable both in light of the equity principle and of some research findings. For example, Shani-Zinovich and Zeidner (2009) report that gifted students in homogeneous (ability-level) classes demonstrated a higher degree of commitment than gifted students in heterogeneous classes. Homogeneous classes, however, can have a negative effect on students' self-evaluation, self-esteem, and emotional environment

In the light of the debate on ability grouping the following question demands careful and systematic investigation: What type of ability grouping is the most effective for mathematically gifted students?

3.3 The centrality of mathematical challenge for the realization of mathematical potential

A mathematical challenge is an interesting and motivating mathematical difficulty that a person can overcome (Leikin, 2007). Many authors recognize the centrality of mathematical challenge for the realization of mathematical promise and as a characteristic of the activities in which gifted mathematicians are involved. The importance of mathematical challenge, the approaches in teaching challenging mathematics, and the role of mathematical challenge in school curricula are analyzed from the international perspective in Barbeau & Taylor (2009). Taylor (2009) and Applebaum & Leikin (2007) analyzed types of mathematical challenges for school mathematics classrooms and stress the importance of teachers' mathematical, meta-mathematical and pedagogical knowledge associated with teaching challenging mathematical tasks. Movshovitz-Hadar and Kleiner (2009) consider mathematical challenge as one of the definitive conditions of mathematical courage that advances mathematics as science. They hypothesize that understanding of the underlying mechanisms of mathematical courage can shed light on the ways in which gifted students can be taught. Sheffield (2009) suggests ways in which mathematically promising students can be challenged, and stresses that challenges for students are differentiated according to their mathematical content knowledge, background, and interests.

Mathematical challenge is a necessary condition for realization of mathematical potential. It can appear in different forms in mathematics classrooms. There can be *proof tasks* in which solvers must find a proof, *defining tasks* in which learners are required to define concepts, *inquiry-based tasks, and multiple-solution tasks.* Mathematical challenge depends on the type and conceptual characteristics of the task, for example, conceptual density, mathematical connections, the building of logical relationships, or the balance between known and unknown elements. From the research perspective some questions can be interesting for the future investigation: What are the types of challenging tasks more appropriate for mathematically gifted students? What challenges better develop mathematical creativity? For example, what is the relationship between Olympiad tasks and students' mathematical creativity?

3.4 Teachers and teacher education in the education of mathematically talented students

The last and certainly not least important issue in the education of mathematically talented children and adolescents is the teacher's role in mathematics classroom, their ways of teaching and teacher preparation for the education of the gifted.

According to Brousseau's (1997) one of the teacher's central responsibilities is the devolution of good (challenging) tasks to learners. It is almost obvious that teachers ought to provide each and every student with learning opportunities that fit their abilities and motivate their learning. Sheffield (2009) maintains that teachers have to challenge students who are ready to move to a higher level, and provide hints to students who may be frustrated. Mathematical challenges directed at students' development usually entail scaffolding provided by a teacher. Consequently in Leikin (2009a) I recommend hanging the following motto on the door of all mathematics classrooms: *Exercises for homework – challenges for the classroom* (ibid. p. 405).

One way of helping teachers to use challenging mathematics in their classrooms is to provide them with appropriate learning material (e.g., a textbook) and make a large number of challenging tasks available to them (Barbeau & Taylor, 2009). However, merely providing teachers with ready-to-use challenging mathematics activities is not sufficient for the implementation of these activities. Teachers must be aware and convinced of the importance of mathematical challenges, and they should feel safe (mathematically and pedagogically) when dealing with this type of mathematics (Holton et al., 2008).

Furthermore, teachers must have autonomy in employing this type of mathematics in their classrooms (Krainer, 2001; Jaworski & Gellert, 2003). They should be able to choose mathematical tasks themselves, create these tasks, change them so that they become challenging and stimulating, and, of course, must be able to solve the problems.To fulfill these conditions, teachers' mathematical knowledge should allow them to cope with challenges presented to their students and their pedagogical knowledge and skills should support scaffolding that teachers provide to their students (Evered & Karp, 2000; Even et al., 2009). Moreover, teachers have to be committed to the purpose of talent development and believe that this purpose is valuable. Last but not least important, teachers have to be provided with multiple opportunities to advance their knowledge, to develop commitment and belief.

Many more questions, such as who can be a teacher of mathematically talented students and how these teachers should be educated are open for systematic research. The following questions need our attention: Should the teachers of gifted be gifted? Should the teachers be creative in order to develop students' creativity? How teachers' creativity can be characterized both from the mathematical and from the pedagogical points of view? What are the desirable qualities of teachers' knowledge, beliefs and personality that make them creative and gifted teachers?

CONCLUSION

Education of mathematically talented children and adolescents is an extremely complex field. People hold different views over the education of gifted which are strongly dependent on their personal experiences and histories related to the education of the gifted. This is true of school students, parents, teachers, teacher educators, educational researchers and educational leaders and managers. Learning opportunities are the most critical factor for the realization of human intellectual potential. Leikin (2009a) pointed out the components that are crucial in developing the students' mathematical potential:

- Parental support (not pressure) – both financial and intellectual;
- Availability of special settings and frameworks for highly capable students in schools and out of schools;
- The necessity of involving technological tools that promote mathematical creativity in students and support teachers' attempts to scaffold students mathematical inquiry;
- Mathematical challenges as a central characteristics of learning environment that develops creativity and promotes mathematical talent;
- Teachers' proficiency in choosing and managing mathematical challenges.

In this paper I argue that each of these components should be a subject for the systematic research in mathematics education.

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