Vectors in Climbing

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Abstract: *In this article, the work on mesospace embodiments of mathematics is further developed by exploring the teaching and learning of vector concepts through climbing activities. The relevance and connection between climbing and vector algebra notions is illustrated via embedded digital videos.*

Keywords: climbing and mathematics; digital media; egocentric vs allocentric representations; embodied mathematics; flow; mathematisation; mathematical archaeology; mathematics and physical education; meso space; meso space representations; reflective research practice; vectors; teaching and learning geometry

Introduction

The teaching of vectors is not an easy task for mathematics teachers. It is difficult to illustrate the concepts of vector calculus exclusively by means of blackboard and chalk (Perjési, 2003), and it also is difficult to succeed in the teaching of vectors (Poynter & Tall, 2005). The literature on the teaching and learning of vectors is very scant in mathematics in comparison to physics education. The dearth of studies addressing student difficulties in vector concepts seems troubling given the basis it forms for vector Calculus, applied analysis and other areas of mathematics. Fyhn (2007, 2010) used children's experiences with physical activities and body movement as a basis for the teaching of angles. In this previous work the focus was on angles in a climbing context. One idea behind the work with angles in a climbing context for primary school students was to lay the groundwork for further work with vectors in upper secondary school (ibid.). The Norwegian curriculum introduces vectors as part of the mathematics syllabus in the second grade of upper secondary school.

A climbing video can introduce vectors without a blackboard and chalk, and such a video might aid in students' understanding. In addition, a video has the possibilities of freezing the picture and drawing vectors on it, in order to guide the watcher's focus. This paper presents and analyses the climbing-and-vectors-video "Vectors in Climbing" found at http://ndla.no/nb/node/46170?from_fag=56.

-Click link to activate video stream-

The actors in this video are two skilled climbers, Birgit and Eirik, who are second year students at upper secondary school. Both had chosen theoretical mathematics as a subject at

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school. To research whether this video may be a useful contribution to the teaching of vectors, the following question was posed

> How are vectors introduced in the video "Vectors in climbing"?

The choice of context

Students should learn mathematics by developing and using mathematical concepts and tools in day-to-day situations which make sense to them (Van den Heuvel-Panhuizen, 2003). The fantasy world of fairytales as well as the formal world of mathematics might function as contexts for mathematical problems. The point is that the students have to experience the context as "real" in their own worlds of thoughts and imagination (ibid.).

Climbing is an activity that can be carried out by most young people given the opportunity and basic instruction. Several places in Norway have indoor climbing walls at a reasonable distance from the upper secondary schools. Many classes include students who climb regularly as a recreational activity. Students with some climbing experiences are familiar with the climbing context, and they will be able to imagine what goes on in the video. Those with no climbing experiences will to some extent be able to imagine what goes on. Most people have experienced climbing trees or rocks as children.

The concept of 'flow' was introduced by Csikszentmihalyi (2000). *Flow* is the holistic feeling one gets when one acts with complete involvement. Humans seek *flow* for its own sake, and not as a means for achieving something else. Professional athletes have referred to the notion of flow during moments of peak performance and described it as a feeling that transcends duality.

> Achievement of a goal is important to mark one's performance but is not in itself satisfying. What keeps one going is the experience of acting outside the parameters of worry and boredom: the experience of flow. (ibid. s. 38)

One quality of *flow* experiences is that they reveal clear and unambiguous feedback to a person's acting. Climbing, chess and mathematics are examples of *flow* activities (ibid.). The climbing video offers vectors as an integrated part of an activity which is experienced as exciting and fun by some young people. In the video Birgit and Eirik claim, "… in climbing you always have to … have fun!"

Bodily geometry[2]

Human mathematics is based upon experiences with our bodies in the world, and therefore the only mathematics we are able to know is the mathematics that our bodies and brains allow us to know (Lakoff & Núñez, 2000). When someone presents you with an idea, you need the appropriate brain mechanism to be in place for you to (hopefully) understand it, and then learn it or reject it. Consequently, "mathematics is fundamentally a human enterprise arising from basic human activities" (ibid. p. 351). This perspective matches Freudenthal (1973), who claimed that geometry is grasping space. "Space" here means the space in which the child lives, breathes and moves. The goal of teaching of geometry is that the students are able to live, breath and move better in the space (ibid.).

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[2] The author uses the term "bodily geometry" to refer to experiences that involve the entire body as opposed to focusing on specific parts or motions as done by those working in semiotics.

Bodily geometry is geometry teaching that builds upon the students' own experiences with their complete bodies present in a three dimensional world. The goal of bodily geometry is a) that the students are able to understand the outcomes of their own bodily actions, and b) that the students are able to use geometry as a tool for living and moving better in particular contexts. The two central goals of the video are i) that students are able to use climbing as a resource for their understanding of vectors, and ii) that students may use their knowledge about vectors to analyse what they do and then improve some of their movements. The last goal mainly concerns students who are climbers.

Different conceptions of space

According to Lakoff and Núñez (2000) space is conceptualized in two different ways through the history of mankind. We cannot avoid the first one, the 'naturally continuous space'; "It arises because we have a body and a brain and we function in the everyday world. It is unconscious and automatic" (ibid., s. 265). Climbing takes place in this *naturally continuous space* where coordinates and axes do not exist. Descartes' metaphor, 'numbers-as-points-on-a –line' lead to the metaphor 'space-as-a-set-of-points', which is ubiquitous in contemporary mathematics. Even though it takes special training to think in terms of the *set-of-points* metaphor, it is taken for granted throughout contemporary mathematics (ibid.): One aim in the Norwegian mathematics syllabus LK-06 was "to visualize vectors in the plane, both geometrically as arrows and analytically in co-ordinate form." (KD, 2006, p. 3), and one more aim was "to calculate and analyze lengths and angles to determine the parallelity and orthogonality by combining arithmetical rules for vectors" (ibid.). The video concerns vectors in the *naturally continuous space* and no calculations take place. A large part of the video focuses on how vectors may be added. A basic understanding of this is necessary before the students are able to succeed in vector calculus.

Berthelot and Salin (1998) subdivided space into three different categories with respect to size; microspace which corresponds to grasping relations, mesospace which corresponds to spatial experiences from daily life situations, and macrospace which corresponds to the mountains, the unknown city and rural spaces. Poynter and Tall (2005a; 2005b) used the term physical activity if the students move an object they hold in their hand, even if they are sitting on a chair performing a micro space activity. This is not bodily geometry; bodily geometry takes place in meso space. The vector video presents bodily geometry, because the actors move their own bodies in meso space.

The mathematical abstraction 'line segments' are connected with 'long objects' via the phenomenon of the rigid body. And a rigid body remains congruent with itself when displaced (Freudenthal, 1983). A climber is familiar with her or his own limbs and body. One point of climbing is not to fall off the wall; how to place the body and how to make it rigid. Freudenthal further claimed

> I am pretty sure that rigidity is experienced at an earlier stage of development than length and that length and invariance of length are constituted from rigidity rather than the other way around. Rigidity is a property that covers all dimensions while length requires objects where one dimension is privileged or stressed. (ibid., p. 13)

In bodily geometry a rigid body is an important part of 'length'; a long arm is able to reach longer than a short arm. And if you stand on your toes you are able to reach a little further.

Climbing: a natural alternation between *egocentric* and *allocentric* frames of reference

Berthoz (2000) referred to 'personal space', 'extra personal space' and 'far space', where *personal space* in principle is located within the limits of a person's own body. According to Berthoz (ibid.) the brain uses two different frames of reference for representing the position of objects. The relationships between objects in a room can be encoded either 'egocentrically', by relating everything to yourself, or an 'allocentric' way, related to a frame of reference that is external to your body. Only primates and humans are genuinely capable of *allocentric* encoding. Children first relate space to their own bodies and the ability of *allocentric* encoding appears later (ibid.). Moreover, "*allocentric* encoding is constant with respect to a person's own movement; so it is well suited to internal mental simulation of displacements" (ibid., p. 100).

When you are trying to ascend a passage of a climbing route, you encode the actual passage *egocentrically* within your *personal space*. But when you stand below a climbing route considering whether or how to ascend it, you exercise *allocentric* encoding in *extra personal space* by considering how the route's different elements and your body relate to each other.

The climber Fredrik explains how this takes place
http://www.uvett.uit.no/iplu/fyhn/fredrik/fredrik_forklarer.html

-Click link to activate video stream-

Climbing can offer students good possibilities for moving back and forth between *egocentric* and *allocentric* representations. While one climber is struggling with a problem, other fellow climbers are often keen on trying to solve the problem themselves. Through their *allocentric* encoding they may imagine how to solve the problem. Next, they make use of *egocentric* coding in their attempts to solve the problem.

Mathematical archaeology

One idea of the video was to start with an activity which many students find exciting and fascinating, and then search for some of the mathematics within the activity. This is done through mathematical archaeology (Fyhn, 2010). A mathematical archaeology is an educational activity where mathematics is recognized and named. This involves being aware that some activities are in fact mathematics (Skovsmose, 1994). "An aim of a mathematical archaeology is to make explicit the actual use of mathematics hidden in the social structures and routines" (ibid., p. 95). The term 'archaeology' refers to a systematically 'un-earthing' (Torkildsen, 2006) of something hidden. In this case vectors are *un-earthed* from three climbing situations. In Fyhn (2010), a detailed description of how angles concepts were un-earthed was given. This idea is now extended to the realm of vectors.

The idea was to offer a more thorough presentation of vectors in one particular context, instead of making a presentation of all aspects of vectors. So the intention was not to start with the learning goals in the syllabus, and then search for some teaching that will fulfill these demands. Climbing is an activity, which requires full concentration. Every move concerns problem solving; how do you position your body so that your hands and feet do not loose the grip of the holds? Climbing does not necessarily concern bodily geometry, because most climbers do not reflect on any mathematics while climbing. By performing mathematical archaeology on some climbing situations, bodily geometry can be the focus.

Vectors might be a useful tool for explaining what goes on in some climbing situations, but it is not given that these situations will constitute a proper basis for the teaching of vectors. This is one limitation of this approach.

Intuitive and formal understanding

Fischbein (1994) claimed, that mathematics should be considered from two points of view; as formal deductive knowledge as found in high-level textbooks, or as a human activity. He pointed out that the ideal of mathematics, as a logically structured body of knowledge, does not exclude the necessity to consider mathematics as a creative process. Mathematics is a human activity, which is invented by human beings.

Fischbein (ibid.) considered the interactions between three basic components of this human activity: 1) The formal aspect: Axioms, definitions, theorems and proofs, 2) The algorithmic component: We need skills and not only understanding, and skills can be acquired only by practical, systematic training, 3) Intuition: Intuitive cognition, intuitive understanding, intuitive solution. Intuitive cognitions may sometimes be in accordance with logically justifiable truths, but sometimes they may contradict them. The point is that "it is the intuitive interpretation based on a primitive, limited, but strongly rooted individual experience that annihilates the formal control or the requirements of the algorithmic solution, and thus distorts or even blocks a correct mathematical reaction" (ibid., p. 244). A climbing video may offer a connection between formal and intuitive understanding of vectors, as long as the students have an expectation of what will happen in the video; a connection between the paragraphs 1 and 3 above.

The teaching of vectors

In Norway vectors is not part of the compulsory mathematics syllabus. The students, who choose a theoretical perspective on the mathematics subject, are introduced to vectors in their second year at upper secondary school. Those who choose to learn physics, meet vectors there too, later in the same school year. The national syllabus (KD, 2006) included the following competence aims about vectors for the second year of upper secondary school mathematics

> The aims of the studies are to enable students to …
> - visualize vectors in the plane, both geometrically as arrows and analytically in co-ordinate form
> - calculate and analyze lengths and angles to determine the parallelity and orthogonality by combining arithmetical rules for vectors (ibid., p. 3)

No research report is found on the teaching of vectors in Norwegian schools. Actually, there are not many research reports to be found regarding the teaching of vectors where computer software is not used. One reason for this might be Perjési's (2003) claim, that it is difficult to illustrate vector calculus by blackboard and chalk. Poynter and Tall (2005a; 2005b) however, represent an exception; they challenged the accepted British ways of teaching.

The British teaching culture uses practical approaches to practical problems (Poynter & Tall, 2005a); vectors are introduced in practical situations in physics. But students find the teaching difficult. The mathematics teachers are aware of their students' problems, but the tendency is that they focus on what the students do incorrectly, without saying why (Poynter & Tall, 2005b). A possible solution to this problem is: One important goal of the teaching is that "the parallelogram law, the triangle law and the addition of components of vectors are all seen as different aspects of the same concept" (ibid., p. 132). This goal is not included in the

Norwegian syllabus. Poynter and Tall (ibid.) pointed out that significant distinctions of meaning in different contexts affect the way students think; they apply the triangle law for displacements and the parallelogram law for forces. Their suggested way to reach the above goal was that the students first construct the essential meaning of 'free vector' and then apply this meaning to vectors in different contexts. The overall goal is "to create conceptual knowledge with a relational understanding of the concepts, rather than a procedural knowledge with an instrumental understanding of separate techniques" (ibid., p. 133). In the vector case, a relational understanding of concepts is interpreted to mean a) relations between different aspects of vectors and b) relations between vectors and other concepts. Other relevant concepts are magnitude, direction and angle.

A conceptual knowledge with a relational understanding of vectors can constitute a proper basis for students on their way to achieve the competence aims in the Norwegian syllabus. The Norwegian syllabus to a large extent left it up to the teachers to find out how the students may achieve the syllabus' aims.

Method

The video "Vectors in climbing" presents vectors in three climbing situations. The actors are two young skilled climbers who were also highly successful in mathematics at school. The climbing situations were analysed with respect to vectors, and the result is bodily geometry, which concern vectors. Therefore the video was categorised as mathematical archaeology (Skovsmose, 1994) with regards to the climbing situations. Some apparently hidden vectors in climbing were uncovered by being identified, named and described. The archaeology was carried out by an un-earthing (Torkildsen, 2006) of vectors that are hidden in these climbing situations. The analyses will enlighten how vectors were presented throughout the video.

The analyses in this paper was performed with respect to Poynter and Tall's (2005b) goal; a conceptual knowledge with a relational understanding of the vector concept. This means that a) the triangle method, the parallelogram method and the decomposition of vectors should be treated as three aspects of the vector concept, and that b) relations between vectors and other concepts are in focus. Furthermore, the video was analysed with respect to c) the treatment of *free vector*, due to Poynter and Tall's (2005b) treatment of this aspect. The two actors' responses to the video was also analysed with respect to connections between intuitive and formal understanding. After they had watched an early version of the video, Birgit and Eirik were interviewed and asked their opinion about the relevance of the video for teaching as well as for their own climbing. The Data in this study is the video "Vectors in climbing" and the researcher's hand written notes of Birgit and Eirik's answers to the interview questions.

Relevance

As stated previously, vectors are introduced to the Norwegian students who choose to study theoretical mathematics in upper secondary school's second grade. Typically students claim that vectors are strange, abstract and difficult, and have nothing to do with the mathematics they know. Birgit from the video explained that she met vectors in a meteorology context at school, and that was difficult because she knew nothing about meteorology. Whether a student considers a syllabus subject important or not, depends on whether the student feels that it concerns her or his overall life situation (Mellin-Olsen, 1987). For the teacher this means to focus on the following questions,"…how does the lesson relate to her students' conception of the important totalities of their world, and how can this lesson eventually transform this totality?" (ibid., s. 33). Consequently the choice of context and teaching method is important in the introduction of new subjects in mathematics. The video intended to relate to what climbing youths consider as important totalities in their world.

A demonstration video like "Vectors in climbing" does not have the same value as it could have if the students were able to perform the climbing themselves. One advantage of the video is that students might watch and re-watch it at their own speed. According to Valdermo (1989) the teaching advantage from a demonstration may increase if it is made close to the intentions of a student experiment. The benefit of a demonstration is that the teacher is given a possibility of steering and controlling what goes on.

Mathematics and physics

In a climbing context vectors means 'forces'. And forces are part of the physics syllabus for Norwegian upper secondary school students. However, mathematics and physics are related disciplines.

> …we see them rather as related disciplines that attempt to mathematise and physicalisise our surrounding world, i.e. to describe phenomena in physical and mathematical terms in order to act and deal with them in a sensible way. (Doorman, 2005, p.3)

To ensure the quality of the physics part of the climbing video, two professionals were consulted: one retired physics teacher and one physicist. The teacher has 90 ECTS credits in physics, a master's degree in meteorology, and thirty years' experiences as an upper secondary physics teacher, but no climbing experiences. The physicist was a skilled and experienced climber with a range of experiences from the Yosemite Valley to Norwegian crags and local indoor walls. He held a Ph.D. in physics.

Analyses of the video

The video presents descriptive mathematics; there are no tasks with numbers for calculations. The last part, focus five, includes some problem solving tasks. Skovsmose (1994) indicated that teachers prefer applied mathematics to descriptive mathematics. Descriptive mathematics might contend no tasks, and that could be the reason why many teachers prefer to work with applied mathematics. There is a risk that some teachers (and maybe students, too) dislike the video because most of it presents descriptive mathematics. So when the video is tested, it s necessary to be aware about this.

Poynter and Tall (2005b) underlined that one teaching goal is to create conceptual knowledge with a relational understanding of the concepts, rather than a procedural knowledge which includes no more than an instrumental understanding of separate techniques. It is of great importance that the "triangle law", the "parallelogram law" and the addition of components of vectors all are seen as different aspects of the same concept. In the climbing video these three aspects are closely connected. But the video does not focus on what a vector is, except for claiming that a vector has a magnitude and a direction. Poynter & Tall (ibid.) described one possible way of guiding the students towards grasping a *free vector* concept in order to enlighten what a vector is. For this purpose they start out with an activity where the students physically displace some figures. The video's weakness is its treatment of the very basic part of vector teaching. While Poynter and Tall (ibid.) described an inductive approach, the video is trapped by a deductive approach: repeated definitions.

The video's strength is that it underlines relations between different aspects of vectors and relations between vectors and angles. The triangular method and the parallelogram method are presented in two of the situations, while decomposition of vectors is presented in all three situations. Relations between vectors and scalars are briefly presented in the introduction, where a picture of a thermometer is assisted by a short text: "A magnitude without direction is called a scalar. It does not make sense to speak of the direction of a temperature". But the video does not mention that the length of a vector also is a scalar.

Relations between vectors and angles are presented in all three situations, and in addition, this part includes several problems posed to the audience. This indicates a need for a profound knowledge about the direction of vectors. According to Fyhn (2007) angle is a central element in the knowledge of vectors. Mitchelmore and White (2000) claimed, that the easiest angles to grasp are those where both of the legs are visible. In a climbing context there are a variety of angles where both of the legs are visible. There are situations where only one leg is visible, too. When the climbing-and-vectors video focuses on one particular angle, the picture is frozen and the angle is drawn on the picture, in order to a) underline which angle is in focus and b) make both of the legs visible. The video also challenge the students in imagining the connection between a change in angle and a change in what happens in the video. This is done in order to focus on the effect of different actions, just as Poynter and Tall (2005b) pointed out.

Vectors having the same length and same direction are called equivalent (Anton, 1981). According to Poynter and Tall (2005b), the *free vector* as a manipulable concept correlates with success on a delayed test about vectors and "The problem is how to encourage the students to construct the flexible concept of *free vector*" (ibid., p. 133). This is a central element in the work with vectors. The point with a transformation is not the transformation itself, but its effect

> The *effect* of a physical action is not an abstract concept. It can be *seen* and *felt* in an embodied sense. The idea is that, if students had such an embodied sense of the effect of a translation, then they could begin to think of representing it in terms of an arrow with given magnitude and direction. (ibid. p. 133)

Common examples of climbers' experiences with the use of their own bodies are: the effect of what happens, when a climber falls and the rope is tightened, or how a climber may place her or his body so that one of her or his feet can stand on a vertical surface. Climbers have a variety of experiences with *egocentric* and *allocentric* encoding of situations like these, while non-climbers to some extent might imagine them. The intention of the climbing video is to offer the students a possibility to imagine and to feel what happens through the different sequences, by an alternation between *egocentric* and *allocentric* encoding of *meso space* activities.

Birgit and Eirik's responses to the video

The first version of the video was shown separately to Birgit and Eirik. Eirik expressed that he grasped most of it. Birgit watched the video in silence, and said that some of it was still difficult. Some other young climbers, who occasionally were present, claimed that what followed after focus one, was theoretical and difficult to understand. This indicates that the video probably does not reach those who do not understand what a vector is. The video presents vectors in situations that the students may imagine. But on this occasion, the deductive approach to what a vector is, confirmed these young climbers' beliefs that mathematics was too difficult to them to grasp.

Birgit and Eirik were interviewed separately immediately after they had watched this version of the video. Concerning the video's relevance, Birgit claimed

> If I had watched this video when I was working with vectors at school, I certainly would have been thinking more about it in my climbing. When I worked with vectors at school, I needed quite a long time to grasp what a vector really is and how to use it in practical situations. If I had seen the video and had taken part in the production of it while I was working with vectors at school, it probably would have been easier for me to understand.
>
> We learned a lot about vectors through meteorology, but I do not know anything about meteorology. But if you learn about vectors through more ordinary things, then you might connect it to things you already know.

To Birgit, climbing is an ordinary thing. Her claims are interpreted to mean that she is not able to imagine what meteorology is, and consequently it is difficult for her to grasp what a vector is. Van den Heuvel-Panhuizen (2003) pointed out that it is important to present the mathematics in contexts that the students are able to imagine, just to avoid experiences like the one described by Birgit.

Birgit and Eirik were also asked which of the sequences they preferred, and they were also given the opportunity to suggest other possible sequences that might be relevant for the video. They both liked the *foot work* best, Birgit because she thinks a lot about it in her climbing. In addition she believes this sequence is the easiest one to grasp. Eirik said, regarding *foot work*, "This is something that I do all the time. This sequence is directly transferable to climbing, so this one was most useful to my climbing… I may use mathematics as a tool to find out how to do things the easiest way." In addition Eirik argued why he believed that all three sequences were good. Both Birgit and Eirik are interpreted to mean that they believe *foot work* to be the sequence with most relevance to climbing. Both are interpreted that a) their intuitive and their formal understanding (Fischbein, 1994) of vectors were connected and b) their *allocentric* coding of the sequence is used in an *egocentric* coding (Berthoz, 2000) of their own climbing. Regarding *pendulum*, Eirik claimed, "The circular arc in relation to the fall - I did not think about that before. So, purely mathematically, this sequence was the most thought-provoking to me." Birgit, on the other hand, was interpreted to feel that this sequence presented too much at a time: "I would like to have watched *pendulum* some more times". Her answer may also be interpreted to mean that this use of video is useful, because the watcher may stop the video and watch each sequence a desired number of times.

Birgit claimed that the video was long enough with three sequences. But she suggested that there might have been some more sequences related to the climbing of overhangs, but that this maybe could turn out to be difficult to present in a video. Birgit's suggestion will be left as an idea to teachers who want to let their students do project work concerning vectors. Eirik had one definite suggestion: "One thing I remember from school was that many students struggled with addition and subtraction of vectors. Consequently, that issue might be included." Eirik's comment here was of great importance because addition of vectors was included in the video at this moment. His answer was interpreted to mean that the video treated this issue too superficially. Birgit and Eirik responded differently to the question about whether climbing might be useful for the understanding of climbing. Birgit doubted it:

> I don't know. If you start explaining by vectors it probably becomes confusing. You have to know these things, even if you don't know that it is vectors. You have to know where the rope is oscillating in order to have a safe fall. And every climber knows that you must not stand far from the wall while belaying.

Eirik was more positive:

> … when I watched *foot work,* I thought that I might use this elsewhere as well. It is difficult to say anything definite about it, because it goes in and out of my consciousness. Earlier I performed things because I knew it would become easier. But now I reason about which vector that makes it easier.

Their different answers to this question may be interpreted to mean that Eirik has a better understanding of vectors than Birgit, and that he is able to see more possibilities. He shows a clear analytic relation to vectors when it comes to climbing. Birgit's point is that you can climb at a high level without knowing anything about vectors. The response from the other young climbers who watched the video together with Birgit, indicate that the video does not represent a good introduction to vectors. The video presents relations between different aspects of vectors, but that is of little help to those who do not know what a vector is! Some weeks after the interview, Birgit and Eirik showed the video to some classes in lower secondary school. These students found vectors difficult. Some days later Eirik claimed that he believed no one but Birgit and him would learn anything from the video. This claim is interesting, and it supports the idea of using the video as an example of how to perform mathematical archaeology. Students might use the video as a basis for interdisciplinary project work in mathematics and physical education. The students may then work in groups according to what physical activity they prefer to enlighten.

Responses from Birgit and Eirik's School

The video was presented to Birgit and Eirik's mathematics class at school, after they had completed their work with vectors that year. Their mathematics teacher was present in addition to the school's headmaster and the researcher. When the video ended, the class applauded spontaneously, and their comments were positive. Maybe they acted positively just because they got some attention, or maybe the acted positively because they could relax and watch some of their classmates. The first comment was that the examples were understandable. In the following school year some of the teachers planned to use the video as part of their mathematics teaching. Due to some technical difficulties the video could not be used there. But one of the teachers showed it as part of the physics teaching for Eirik's class. Some weeks before these students' final exam at upper secondary school, the researcher met one boy from this class by chance. The boy asked "You are the one who made the vector video, aren't you?" The question was verified, and followed by a return question about whether he thought the video was of any use. The boy answered that he did not find the video particularly useful before he started working with forces as part of the physics subject. And when he was preparing for his final physics exam, he found the video both interesting and informative. This supports the results from the analyses, that the video has a missing link somewhere between focus one and focus two; the video does not provide the students with an introduction to addition of vectors.

Summing up

A search for the word 'vector' in the mathematics education research literature gives few hits except for those where the context is 'computer based environments'. One reason for this might be Perjési's (2003) claim, that it is difficult to illustrate the concepts of vector calculus exclusively by means of blackboard and chalk. The video 'Climbing and Vectors' meets this challenge because it is based on *bodily geometry*, and it presents *allocentric encoding* (Berthoz, 2000) of activities that the students can easily imagine. The activities take place in the *naturally continuous meso space*, and the students are able to imagine what goes on in the video as van den Heuvel-Panhuizen (2003) pointed out. However, the analyses of the video points out an important weaknesses with the video: a lack of a fundamental explanation of what a vector is. The young climbers, who watched the video together with Birgit, claimed the video was abstract, most likely because of the deductive approach to what a vector is. One way of doing this, is as Poynter and Tall (2005b) described; through *free vector*. The video has some strengths; the close relations between a) the presentation of connections between the triangular method, the parallelogram method and additions of components of vectors, b) the demonstration of relations between vectors and angles. The video also offers a presentation of how to perform mathematical archaeology upon one particular context.

One important question remains. How will the video "Vectors in climbing" influence the students' understanding of vectors? Students with no climbing experiences will probably respond differently from the skilled climbers. The mathematical archaeology in the video was performed by the researcher and two climbers; it is far from obvious how other climbers will respond to the video. Climbers and other young active people may use the video as background for how to perform mathematical archaeology on different activities. The video has two goals: i) that students are able to use climbing as a resource for their understanding of vectors, and ii) that students may use their knowledge about vectors to analyse what they do and then improve some of their movements. Eirik gives examples of things he did not think of before watching the video. He also claims that he has started using vectors as a tool for analyzing his climbing. He is interpreted to make use of *allocentric* encoding (Berthoz, 2000) in these analyses. His answers concern both these goals. Birgit answered that the footwork sequence concerned most of what she does in climbing. She may be interpreted to mean that the video suits students with some climbing experience. Mellin-Olsen (1987) pointed out that the teaching should relate to what the students experience as their world, and Birgit supports this

> It is always nice if you are familiar with things without knowing that it is a vector. If you learn about vectors through climbing, and you know a lot about climbing, then it is easier… If you learn about vectors through daily things, then you might connect it to some of your previous knowledge.

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