Algebra for all?

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Déjà vu all over again

In 1957, the director of one of the major math curriculum projects in the UK was quoted in a newspaper as saying: "Up went Sputnik and down came all the pure mathematicians saying we must do sets and be saved". The corresponding message from the National Math Panel Final Report is "Up went the economic performance of China and India and down came all the pure mathematicians saying we must do algebra and be saved".

Not quite all the mathematicians, however. Davis (1999) wrote:

> What is necessary is to teach enough so that the commonplace diurnal mathematical demands placed on the population are readily fulfilled. What is also necessary is to infuse sufficient mathematical and historical literacy that people will be able to understand that the mathematizations put in place in society do not come down from the heavens: that they do not operate as pieces of inexplicable ju-ju, that mathematizations are human cultural arrangements and should be subject to the same sort of critical evaluation as all human arrangements.
> At the risk of sounding like a traitor to my profession, I would say that high school algebra or beyond is not necessary to achieve this goal.

What is algebra *for*?

> "Algebra … the intensive study of the last three letters of the alphabet"
> (source unknown)

On a personal level, I have positive memories of algebra at school as something that could almost always be done routinely and the answers checked (as opposed to problems in Euclidean geometry that were more intellectually challenging). Yet, for the majority of students, the experience of "pawing at symbols" is not fulfilling, and leaves no trace beyond a general negative feeling. It is a troubling experience to sit beside an eighth grader who is vainly trying to remember what to do with an algebraic equation and reflect that several more years of frustration lie ahead for that student.

While I derived reinforcement from being competent at doing it, I have no reflection of any of my technically excellent teachers ever discussing what algebra is *for,* except, of course, the passing of exams and progression to higher levels of the same.

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Kaput (1999) presents what I consider a balanced framework for teaching school algebra. He begins (p. 133) by stating:

> The traditional image of algebra, based in more than a century of school algebra, is one of simplifying algebraic expressions, solving equations, learning the rules for manipulating symbols – the algebra that almost everyone, it seems, loves to hate…
>
> …School algebra has traditionally been taught and learned as a set of procedures disconnected both from other mathematical knowledge and from students' real worlds.

The importance of algebra at a societal level is clear (p. 134):

> … algebraic reasoning in its many forms, and the use of algebraic representations such as graphs, tables, spreadsheets, and traditional formulas are among the most powerful intellectual tools that our civilization has developed. Without some form of symbolic algebra, there could be no higher mathematics and no quantitative science; hence no technology and modern life as we know them. Our challenge then is to find ways to find the power of algebra (indeed, all mathematics) available to all students…

I include this citation, with which I agree (who wouldn't), to make the point that the argument in this paper is not against the worth of algebra, nor against making it accessible and intellectually enjoyable to all students who want it, but against the declaration by fiat that all students will learn algebra, will be required to pass courses in algebra to have good educational and economic opportunities, and – above all – will continue to be taught algebra in a boring fashion.

Kaput (p. 134) proposes the following broad outlines for a more productive approach to the teaching of school algebra:

- begin early (in part, by building on students' informal knowledge);
- integrate the learning of algebra with the learning of other subject matter (by extending and applying mathematical knowledge),
- include the several different forms of algebraic thinking (by applying mathematical knowledge);
- build on students' naturally occurring linguistic and cognitive powers (encouraging them at the same time to reflect on what they learn and to articulate what they know), and
- encourage active learning (and the construction of relationships) that puts a premium on sense making and understanding.

Kaput suggests that there are five forms of algebraic reasoning, namely (a) the generalization and formalization of patterns and constraints, (b) syntactically guided manipulation of (opaque) formalisms, (c) the study of structures abstracted from computations and relations (not limited to generalized arithmetic), (d) the study of functions, relations, and joint variation, (e) a cluster of modeling and phenomena-controlling languages.

I consider it bizarre that Kaput's empirical, curricular, and conceptual work on algebra, based on an unmatched knowledge of mathematics, the intellectual history of mathematics, psychology, cognitive and developmental psychology, epistemology, semiotics, and so on, is not represented in the NMAP report. The failure to reference the review chapter by Kieran (2007) and the work of so many other leading researchers working on algebra within mathematics education is likewise part of the pattern of exclusion referred to in this issue's editorial..

Who needs algebra?

Is it not a fundamental characteristic of an advanced society to have differentiation of roles? Instead of demanding "Algebra for all" why not something like "A great deal of algebra for a few, some algebra for the majority" (admittedly inferior as a sound-bite-sized slogan).

McGregor (2001, pp. 405-406) reported a selection of responses to the question "Have you ever made use of any of the algebra you learned at school?":

> House painter: "Algebra? All that *x* and *y* stuff? I had no idea what it was about. I couldn't do maths at all, even in primary".
> Bus driver: "Never. I could understand some of it in my head but I never knew how to write it".
> Primary teacher: "Algebra? Oh, I remember, if you have 2 and then a bracket and then *a* plus *b*, it's 2*a* and 2*b*. Is that right?
> Lecturer in literacy: "Algebra! I never understood it. I hated maths and was no good at it".
> Journalist: "Never, but I liked solving problems with calculus at school".
> Electrical engineer: "Yes, I work with formulas all the time. that's algebra. I always work with spreadsheets. I use Excel. That's all I need. I know what the answers should look like; that's important".
> Financial mathematician consulting to large corporations: "Not really, not that kind of algebra. I write individual programs for each client. I use APL. Many people doing my kind of work use Visual Basic".
> Information systems consultant: "No. To solve a problem I would write a program, probably in J or Java".

The people who developed Excel, Java, and so on, almost certainly knew plenty of algebra and other advanced mathematics. However, the range of quotations above underlines the point alluded to above about the differentiation of roles in contemporary industrialized societies, wherein so much of mathematics is embedded in tools, and essentially invisible to the users of those tools, As expressed by Skovsmose (2006, p. 325):

> The education systems must ensure a supply of people with competencies according to a matrix that represents society's demands for competencies. Some groups must be well-educated in mathematics; some must be able to operate with certain mathematical techniques; some must be able to read diagrams; some must know the mathematics included in instructions; a great many must know the mathematics necessary for shopping and dealing with payment and bank transactions.

As a response to these needs, "Algebra for all" is simplistic and, essentially dishonest. Arguably the most significant problem, with the severest consequences, is when passing an algebra course becomes a barrier to educational and economic advancement. Why should algebra be necessary for someone who wants to become a musician, a lawyer, a counselor, an English teacher, a politician (you can continue the list)? Whether consciously planned or not, erecting algebra at the entrance to a national educational and economic gated community is an extreme form of political/social engineering. This form of "accessment" gives US education "a way to sort children by race and social class, just like the old days, but without the words 'race' and 'class' front and center" (McDermott & Hall, 2007, p. 11).

The implied dominance of the curriculum by algebra

The assignment given to the panel specifically mentioned algebra in the first of 10 requirements for the report: "the critical skills and skills progression for students to acquire competence in algebra and readiness for higher levels of mathematics" (Presidential Executive Order 13398). In response, the report focuses on algebra to the detriment of a balanced view on mathematics. It is striking how emphasis is put on the teaching of arithmetic, geometry, and combinatorics in relation to their importance in laying foundations for algebra. In geometry, for example, teacing about similar triangles is singled out because they enlighten the graphical representation of linear functions, in particular the invariance of the slope (incidentally, it is only the special case of similar right-angled triangles that is needed for this purpose). The subject of combinatorics, apparently, owes its importance to its relationship to the binomial theorem, not its role in developing applications of probability theory. Thus, the final entry in Table 1 of the Final Report (p. 16) is "combinations and permutations, as applications of the binomial theorem and Pascal's Triangle".

Characterization of algebra in the NMP report

A definitive statement is presented in the Final Report (Table 1, p. 16) as a list of topics that is a subset of what I learnt at grammar school in Northern Ireland nearly 50 years ago, with two exceptions, namely (a) fitting simple mathematical models to data, and (b) combinations and permutations, as applications of the binomial theorem and Pascal's Triangle. This specification is based on a number of comparisons, including the algebra standards in Singapore's mathematics curriculum for Grades 7-10. It is intended as "a catalog for coverage, not as a template for how courses should be sequenced or texts should be written" (footnote 9, p. 15). Nevertheless, an overview of school algebra is presented in the report of the Task Group on Conceptual Knowledge and Skills (pp. 1-6 to 1-15) that is offered as a guide for mathematics teachers and textbook publishers. There are many parts within this explanation that I find odd.

First, in the preamble, it is written (p. 1-6):

> … the discussion of word problems will be somewhat abbreviated. In no way, however, should this emphasis be interpreted to mean that problem solving is considered to be less important than [connections between basic concepts and skills]. Indeed, the solution of multistep word problems should be part of students' routine.

Problem solving should not be equated with word problems. There are many fascinating problems in algebra that are not word problems.

Second, it is firmly stated that the most basic protocol in the use of symbols is "Always specify precisely what each symbol stands for" (p. 1-6). Yet at several points thereafter, new usages of symbols are introduced without specifying what they stand for. Most perplexingly, we find a statement (p. 1-14) "Let X be a symbol". I'm no expert on logic, but it seems clear to me that there is a problem here!

Third, in talking about solving a quadratic equation, it is stated (p. 1-9) that "assuming there is a solution … students deduce what it must be", which sounds odd given that, in general a quadratic equation has two solutions.

A further aspect of the outline of school algebra that strikes me as a little odd is the inclusion of the statement (but not proof) of the Fundamental Theorem of Algebra, namely that every polynomial form of positive degree has a complex root (from which it follows that every polynomial form of degree n can be expressed as the product of n linear expressions). It is further stated (p. 1-15) that "the importance of the theorem justifies that school students learn it and use it even if they will not see how it is proved" (the proof is quite advanced). It is not clear to me how it could be used in the context of high-school algebra.

Assessment

Let me finish with a few comments on the assessment of algebraic thinking, as an area that, above all, shows up the weaknesses of multiple-choice tests, at least as they are currently constructed (they are capable of improvement). For example, consider the following item, cited with approval in the Report of the Task Group on Assessment (p. 6-76):

> Mona counted a total of 56 ducks on the pond in Town Park. The ratio of female ducks to male ducks that Mona counted was 5:3. What was the total number of female ducks Mona counted on the pond?
> A. 15 B. 19 C. 21 D. 35
> Comment: A student has to decide which fractions are relevant.

How might a students solve this problem? The formal approach, as advocated by Wu (2001) is to make use of the property that:

$^{a}/_{b} = {}^{c}/_{d} \Rightarrow {}^{a}/_{(a+b)} = {}^{c}/_{(c+d)}$

where, in this case, $a = 5$, $b = 3$, and $c + d = 56$. That leads to an equation that can be solved, and is, indeed, a way to find the answer. Another way is to think of a grouping of 5 female ducks and 3 males. Dividing 56 by 8 reveals that the ducks may be partitioned into 7 such groupings, hence there are 7 X 5 = 35 female ducks. Even simpler is to realize there are more female than male ducks, so the answer must be more than half of 56, which eliminates all answers but D! As with many multiple-choice items supposedly designed to test algebraic competence, this one is awash with possibilities for false positive responses.

Summary

The main points are easy enough to summarize:

1. I argue against the goal of "algebra for all" on the grounds that, while collectively a cadre of mathematical and scientific specialists is needed for society to operate effectively, most individuals in our society do not need to have studied algebra. This by no means implies that anyone should be denied the opportunity to do so (in an intellectually stimulating way). Moreover, students should be encouraged to study algebra in the spirit of keeping options open, given its status as a gatekeeper to many educational and economic opportunities.
2. There is a need to radically rethink the role of algebra within the mathematics curriculum, as argued by Kaput (1999).

References

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