

Magic Card Maths

Steve Humble¹

The National Centre for Excellence in the Teaching of Mathematics (UK)

Starting lessons with a good story, captures students imagination. The story then leads nicely into a magic card experiment which can be used to help teach the topic. I have used magic with a variety of age groups and found that they ask different questions of the task, and search for different levels of answers.

John Scarne (1903-1985)



John Scarne, who was one of the world's foremost gambling authorities, was not a gambler but has published some of the best and most comprehensive information on gambling and cheating during his reign. His most well known titles include Scarne on Dice, Scarne's Guide to Modern Poker and Scarne's New Complete Guide to Gambling. He was an outstanding and highly skilled magician who specialized in card magic and sleight-of-hand.

¹ Steve Humble (aka DR Maths) works for The National Centre for Excellence in the Teaching of Mathematics in the North East of England (<http://www.ncetm.org.uk>). He believes that the fundamentals of mathematics can be taught via practical experiments. He is the author of the book *The Experimenter's A to Z of Mathematics*, which develops an experimenter's investigative approach to mathematical ideas. Always having had great fun playing with maths, he enjoys teaching this to others.

For more information on DrMaths go to
<http://www.ima.org.uk/Education/DrMaths/DrMaths.htm>

Email: drmaths@hotmail.co.uk

Humble

Scarne had a sharp mathematical mind, but never finished high school. His education came from observing card sharps in local carnival grounds and from a novelty shop owner who made imperfect dice and dubious decks of playing cards.

As well as advising casinos about security Scarne was the technical advisor and doubled for Paul Newman in the classic movie "The Sting" in 1973. In the movie most of the sleight-of-hand demonstrated by Newman actually featured Scarne's hands. He also appeared in the short film "Dark Magic" as well as several commercials.

Scarne's most famous card trick was appropriately titled "Scarne's Aces". The trick involved taking a spectators shuffled deck of cards, performing a series of shuffles himself and then cutting to all four aces.

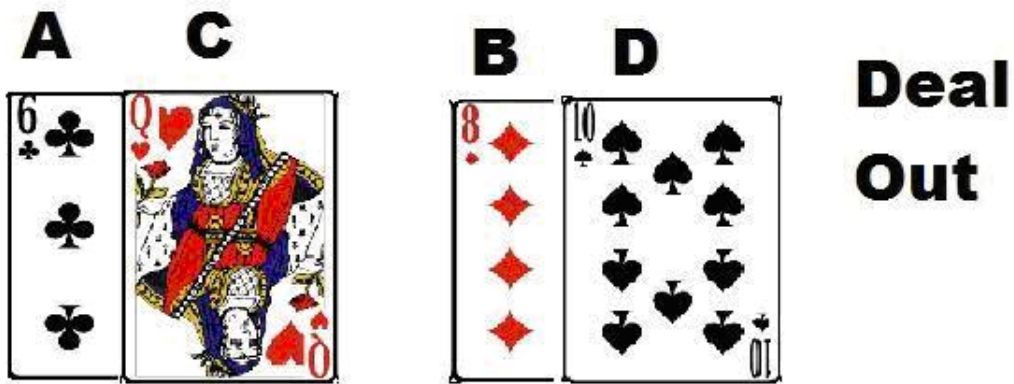
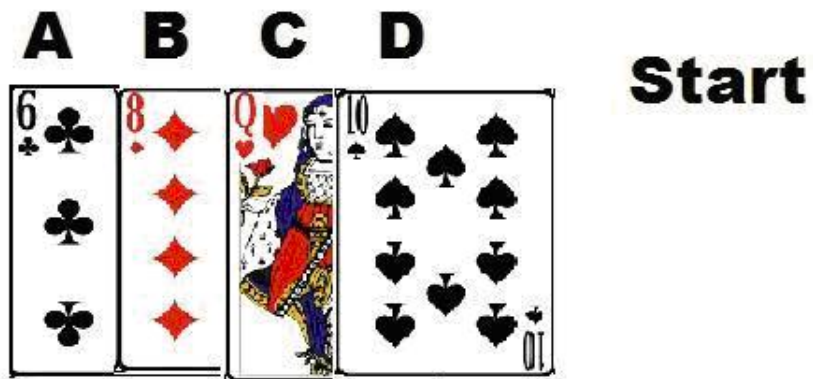
He knew the importance of the long history of magic in relation to his own life and is quoted to have said "All of my adventures and exploits . . . will, of course, be forgotten soon enough," He wrote this at the conclusion of his autobiography 'The Odds Against Me. "Gamblers and magicians come and go but their tricks stay forever"

Several techniques are used to shuffle a deck of cards. While some achieve a better randomization, others are difficult to learn and handle or are better suited for special decks of cards. A procedure called Slide Shuffle is where small groups of cards are removed from the top or bottom of a deck and are replaced on the opposite side. Another common shuffling technique is called a Riffle, in which half of the deck is held in each hand with the thumbs inward, then cards are released by the thumbs so that they fall to the table intertwined. The list of various shuffle types goes on with Hindu, Pile, Chemmy, Mongean to name but a few. See the further references at the end of the chapter for more information.

Lets look at one particular shuffle called the Faro Shuffle or sometimes called the Perfect Shuffle. According to John Scarne this is the most popular shuffle favoured by magicians for its strange properties.

For a Perfect Shuffle the deck of cards is cut precisely in half and interleaved alternately with one card from each half. If you take a normal deck of 52 cards and perform a Perfect Shuffle 8 times you will find that the deck is returned to its original order. You can use the maths behind the Perfect Shuffle to perform the following card trick. It is best to do this trick using 8 or 16 cards, but to explain how it works I will only use 4 cards.

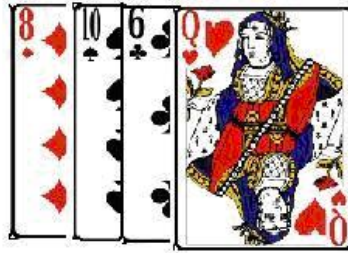
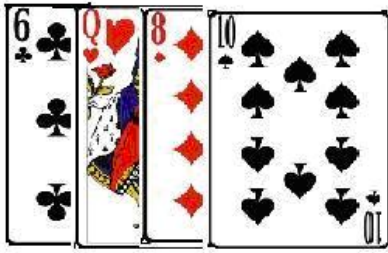
Without telling you, a spectator thinks of a number from 1 to 4, or 1 to 8 if using eight cards, and then counts from card A to that card. Make a mental note of card A, then turns the cards face down and deal alternate cards into two piles. The spectator is then asked which pile their card is in and these cards are put underneath the other two cards.



Turn the 4 cards face down and deal alternately into two piles. Repeat the process of asking which pile the spectator's card is in, then put these cards underneath the other two cards and turn them face down.

Humble

A C B D or B D A C



**Two
Cases**

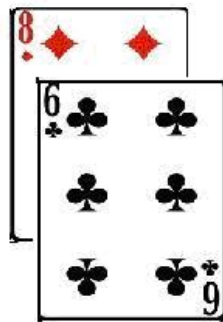
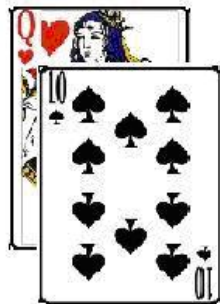
A B

C D or

B A

D C

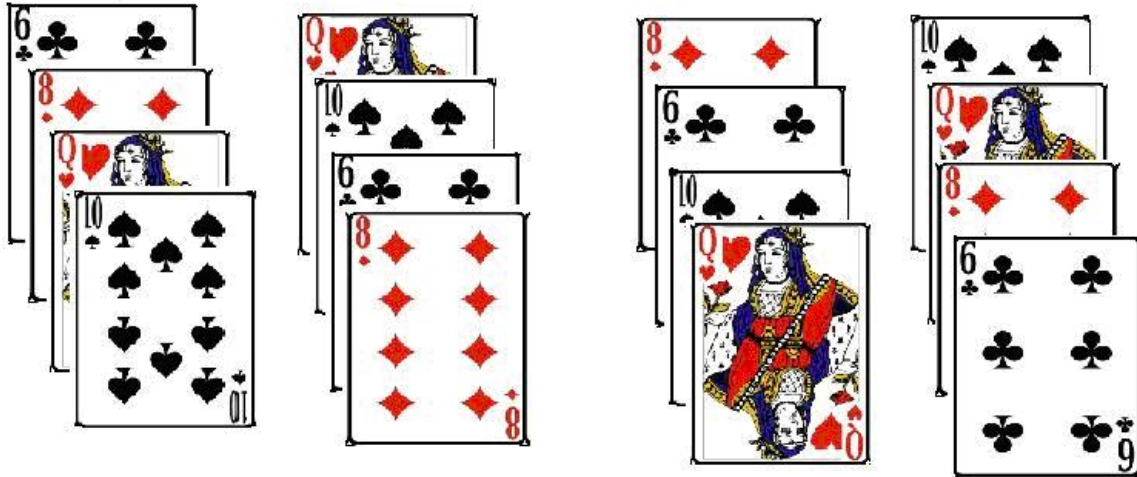
**Deal
out**



Their chosen card is revealed when you turn over the top card. No matter which card they have picked this card will always be at the top. Here are the four possible cases showing each card at the top.

Four possible cases

ABCD or CDAB or BADC or DCBA



If you now count to card A (6 of clubs in this example), which is the card you made a mental note of, you will find that this is their original number 1 to 4. Maths or magic?

To find the spectators chosen card when you do this trick with 8 cards, you will need to do 3 “deal outs” (2^3), with 16 cards 4 “deal outs” (2^4), etc...

To add extra magic to the trick, place the value of card A in an envelope before you start. This will leave the audience stunned, as you have not only found their card and their number, but predicted the value of the card at their number within the deck.

The reason for the number of “deal outs” can be seen by looking at all the possible combinations needed to obtain each card at the top of a pile. From the illustration with four cards you can see you get

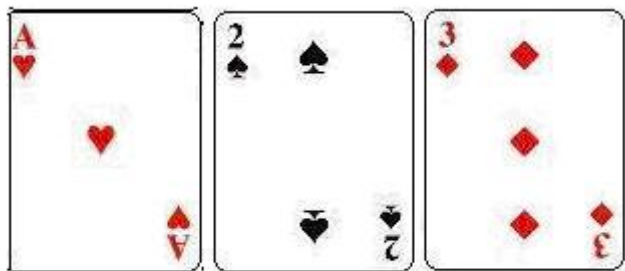
ABCD or BADC and CDAB or DCBA

With eight cards this becomes ABCDEFGH or DCBAHGFE or BADCFEHG and so on generating all eight possible variations of each card in position one and the A card in all possible places.

The maths of the *Perfect Shuffle* is used in everyday life, such as with our home PCs in dynamic computer memories to help improve the storage of data and the interconnections of networks. If you want to find out more about this then see further reading at the end of the chapter.

The Perfect Shuffle card trick relies on the mathematics of arrangements. The next shuffle trick relies on a set of transformations on a small group of cards which result in it returning to its initial form. In mathematics we call this type of transformation invariant, and a number of famous magic tricks use this idea. Bob Hummer was one of the first innovators in this area in the 1940s, and here is one of his card tricks.

Humble



Place any Ace, two and three face up in a row on a table. Turn away and invite the spectator to choose one of the cards and turn it over. They then turn over the other two cards having first switched their positions. You then pick up the cards so that from top to bottom you have the card to your right, the middle card, and the card to your left. Shuffle the cards by cutting the cards to the bottom, one or two at a time. To add to the effect shuffle two cards or one randomly, but keep a secret count and stop once you have moved 4 or 7 or 10 cards, or any addition of three on from 10. Once you have stopped shuffling deal out, face down, the top card to the middle, the second card to your right and the third to your left. In your head imagine these cards are 3, 2 and Ace from left to right. A reversal of the initial set up.

The spectator guesses which card they think is theirs and turns it over, giving no hint as to whether they have guessed right. You either congratulate them on picking their card or say that's not your card and turn over the correct one.

This trick works via these transformations:

Starting with 1 2 3

the spectator picks a card and swaps the other two. You do not know which card is which so let's call them

A B C

You pick up the cards in the reverse order

C B A

Shuffle once, four times, seven times etc and this will always have this effect

B A C

Put the cards back on the table

C B A

3 2 1

If the spectator turns over A, B or C and it matches 1, 2 or 3 then this is their card.

If the spectator turns over A, B or C and it does not match then that card's value tells you what to do:

If C was an Ace then this means the correct card cannot be A, so it must be B.

If C was a 2 then this means the correct card cannot be B, so it must be A.

If B was an Ace then this means the correct card cannot be A, so it must be C.

If B was a 3 then this means the correct card cannot be C, so it must be A.

If A was a 2 then this means the correct card cannot be B, so it must be C.

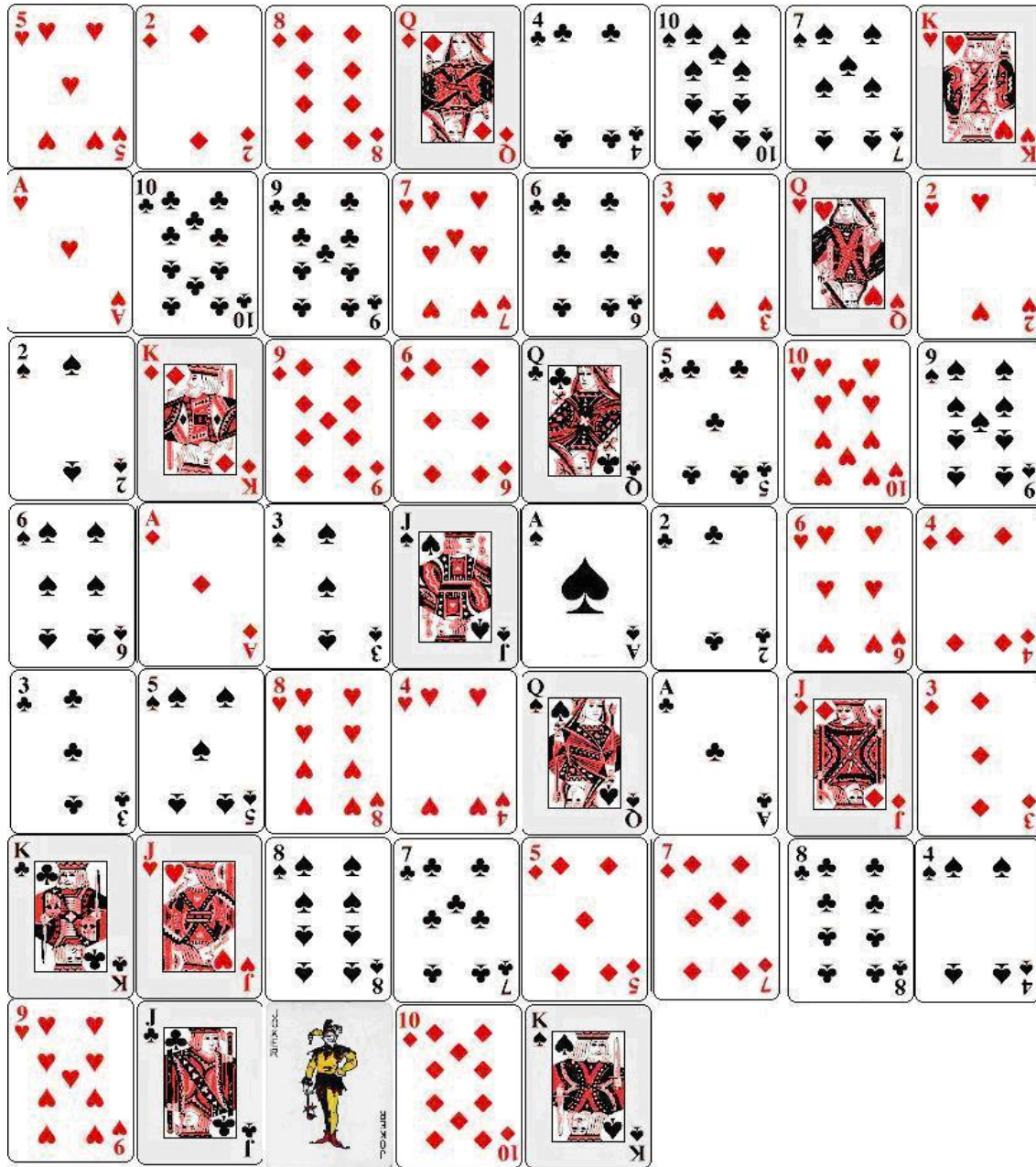
If A was a 3 then this means the correct card cannot be C, so it must be B.

Bob Hummer's prediction trick has little to do with reading the mind of the spectator or seeing the future, but as with all good magic it can show us how we can be confused by what we see. The next card trick has its roots in Probability, and wants us to believe that apparently unrelated chains of events can be doomed to link into sync after a while. Synchronicity is a word coined by the psychiatrist C. G. Jung in his 1973 paper "Synchronicity: An Acausal Connecting Principle". Jung argued that meaningful coincidences occur far more frequently than chance allows for. A great deal of work has now been done in this area. One factor which effects coincidence is the way we experience it. It has been found that the way a story is told can change the degree of surprise. Surprise ratings increase if the same story was told as a potential future event, as opposed to things which have just happened.

So with this in mind, how strange would it be if you and a friend started at two unrelated random points and ended up at the same place? What if you and seven friends started at eight random points and all ended up at the same place? Synchronicity, magic or maths?

Here is a variation on a card trick called Kruskal's Count invented by Martin Kruskal. Ask a spectator to shuffle a deck of cards thoroughly, and then pick a number from 1 to 8. Deal out all the 52 cards, face up, with eight in each of the first 6 rows and four in the last.

Humble



The spectator then starts at the number they have chosen in advance on the top row. So if in the example above they had picked the number 5 in this case it would be the four of clubs. They then count to the next card using the value of the card, e.g. count 4 cards along to the two of hearts. In this example they would then count 2 along to the three of hearts and 3 along to the nine of clubs and so on until they can go no further. Counting any court card they land on as one.

In the above example they will eventually end up at the Joker on the bottom row. No matter which card they start with on the top row they will always end up at the Joker.

Synchronicity, the fact that you and seven friends started at eight random points and all ended up at the same place or is it maths?

This magic trick of randomly dealing out the pack and then picking random starting points will (nearly) always result in one final destination. Once you find this final destination card then mark it with the Joker and ask other spectators to try starting at different points on the top row.

The mathematics for this amazing trick has to do with the fact that the various routes you take to get to the end, cross each other along the way. When these routes have the same cards in their pattern they become one route. As they cross a number of times before getting to the bottom row, the chance is high that they will obtain the same cards. The more cards in the pattern the greater the chance that the routes will join.

Probability can show how high this chance is. With eight cards in the top row, you have a 1 in 8 chance of someone picking the same card as you. The average position of this card in the top row is $(1+2+3+\dots+8)/8=4.5$

The average length of each subsequent jump you make, moving from one card to the next is

$$(1+2+3+\dots+10+1+1+1)/13=4.46$$

The number of expected cards which will be chosen after the first card is $(52-4.5)/4.46=10.65$, which is about 10. The chance you will land on the same card is approximately $1/4.46$

Therefore the chance of missing all the cards is $\left(\frac{7}{8}\right)\left(1-\frac{1}{4.46}\right)^{10} \approx 0.07$

This means that you have around 7 chances in 100 that the trick will not work for all eight cards on the top row, but 93 times out of 100 that it will.

Make your own magic

In this section we will look at the art of making your own card tricks using the mathematics of algebra. Once you have learnt this method you will find a number of other card tricks that follow this pattern.

Take an ordinary pack of cards. Select nine Hearts numbered one to nine and nine Spades numbered one to nine. Ask a spectator to pick one Heart and one Spade, without you seeing which they have chosen. Then ask them to do the following, without disclosing any of the answers:

Take the Heart card, double its value, add one and then multiply the result by five. Add on the total value of the Spade card and ask them to tell you the final answer.

For example if the spectator picked the four of Hearts and seven of Spades, he would have doubled the four to get eight, then added one and multiplied by five to get 45. Finally by adding seven, a total of 52 is reached.

When you are told the final answer, in your head, always subtract 5. The number you have left will show you the two card values. In this case 52 take 5 is 47. So you can give the answer of four of Hearts and seven of Spades.

Humble

Looking at the Algebra shows how this trick works and also lets you make your own personal tricks. Lets say the red Heart card is R and the black Spade card is B.

Double gives	$2R$
Add 1	$2R + 1$
Multiply by 5	$5(2R + 1)$
Add Black Card	$5(2R + 1) + B$
Multiply out the brackets	$10R + 5 + B$

It can be seen that once 5 is taken from this you are left with $10R + B$
The unit digit is the black card and the tens digit is the red card.

Using this idea you can make your own personal card tricks, remembering to end up with tens and units.

For example $2(5R+3) + B$

Five times the red card, add 3 and then multiply the result by 2. Finally add on the total value of the black card.

You would have to take 6 off in your head to find the answer.

Or with a harder trick you could involve division, $3(20R + 18)/6 + B$

Twenty times the red card, add eighteen, multiply by 3 and then divide by six. Finally add on the total value of the black card.

You would have to take 9 off in your head

Why not try and make you own tricks following this idea?

Quick Magic

Here is a simple self working card trick using four Aces in memory of the great John Scarne.

Ask a spectator to pick a number from 14 to 25. Then count out that number of cards from the deck into a pile. Now pick up this pile of cards and deal out four hands as you would if playing a game of cards.

Reveal in whatever way you think best, the amazing chance that four Aces are on the top of each of these piles. What is the chance of that?

The only preparation required for this trick is before you start is to put the four Aces on the top of the deck.

Further references

Magic Tricks, Card Shuffling and Dynamic Computer Memories (Spectrum) by S. Brent Morris
Methods for Studying Coincidences, December 1989, Vol 84, No. 408 ASA by Persi Diaconis and Frederick Mosteller.

Penrose Tiles to Trapdoor Ciphers (W.H. Freeman) by Martin Gardner

Easy Magic Tricks (Sterling) by Bob Large