The Interplay of Processing Efficiency and Working Memory with the Development of Metacognitive Performance in Mathematics

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Abstract: *The present study outlines a specific three level hierarchy of the cognitive system and especially the relations of specific cognitive and metacognitive processes in mathematics. The emphasis is on the impact of the development of processing efficiency and working memory ability on the development of metacognitive abilities and mathematical performance. We had used instruments measuring pupils´ metacognitive ability, mathematical performance, working memory and processing efficiency. We administered them to 126 pupils (8-11 years old) three times, with breaks of 3-4 months between them. Results indicated that the development of each of the abilities was affected by the state of the others. Particularly, processing efficiency had a coordinator role on the growth of mathematical performance, while self-image, as a specific metacognitive ability, depended mainly on the previous working memory ability.*

1. Introduction

There is an increasing consensus that intelligence is a hierarchical and multidimensional edifice that involves both general-purpose and specialized processes and abilities (Demetriou, Zhang, Spanoudis, Christou, Kyriakides & Platsidou, 2005). According to differential theory, individual differences in psychometric intelligence are associated with individual differences in processing efficiency and / or working memory (Engle, 2002; Jensen, 1998). According to developmental theory, developmental changes in thinking are associated with changes in processing speed or efficiency (Kail, 1991), central attentional energy or capacity (Pascual Leone, 2000), or working memory (Case, 1985; Demetriou, Efklides & Platsidou, 1993). In fact, recent research shows that processing efficiency is the developmental factor in regard to the development of working memory and reasoning whereas working memory is a factor of individual differences in regard to the development and functioning of reasoning. That is, changes in processing efficiency open possibilities for changes in working memory and thinking (Demetriou, 2004; Demetriou, Christou, Spanoudis, Platsidou, 2002). The research on the development of metacognitive abilities should be connected with the development of other cognitive abilities, such as speed of processing, control of processing, working memory, attention etc.

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We believe that mathematics does involve some special mechanisms of representation and mental processing which are appropriate for the representation and processing of quantitative relations. At the same time, we also believe that these mechanisms are constrained by the organization and the possibilities of the human brain. Thus, any research about the architecture and the development of mind in respect to mathematics will have to specify the domain-specific processes and functions that it involves, the general potentials and processes of the human mind that sustain and frame its functioning, and their dynamic relations in real time during problem solving.

The relationships among cognitive processes, such as control of processing, speed of processing and working memory with metacognitive processes, such as self-representation, self-evaluation and self-regulation, is one of the issues elaborated by Demetriou and his colleagues (Demetriou & Kazi, 2001; Demetriou et.al, 2002). The present study was motivated by the integrated model proposed by these authors, according to which any change in a particular system affects the functioning of the cognitive processes. For instance, practice in the application of arithmetic operations may make children more aware of their memory limitations. At the same time a change at the metacognitive system influences the functioning of the processing system.

There are important questions that are still debate in psychology and in mathematics education. The present study purports to contribute to the ongoing research on the impact of specific cognitive processes, on metacognitive abilities and on mathematical performance. The purpose of the present study was twofold: First to explore the impact of processing efficiency and working memory on metacognitive processes in respect to mathematics and secondly to explore if the above interrelations tend to change with development.

At the following sections we first present, the concept of metacognition and its relations with young pupils' mathematical performance. Then we connect the metacognitive abilities with the processing efficiency and working memory ability in order to realize its position at the whole cognitive system and its relation with the mathematical performance.

The concept of metacognition in mathematics education

In recent years metacognition has been receiving increased attention in cognitive psychology and mathematics education (Guterman, 2003; Pappas, Ginsburg & Jiang, 2003). The interest has focused on its role in human learning and performance. In modern psychological literature the term metacognition has been used to refer to two distinct areas of research: knowledge about cognition and regulation of cognition. The present study uses the term "metacognition" referring to the awareness and monitoring of one's own cognitive system and its functioning.

Metacognitive knowledge is "knowledge or beliefs about what factors or variables act and interact in what ways to affect the course and outcome of cognitive enterprises" (Flavell, 1999, p.4). The major categories of these factors or variables are person, task and strategy (Flavell, 1987). The "person" category encompasses everything that a person believes about the nature of him/herself and other people as cognitive processors. It refers to the kind of acquired knowledge and beliefs that concern what human beings are like as cognitive organisms. The "task" category concerns the information available to a person during a cognitive enterprise. Thinkers must recognize that different tasks entail different mental operations (Demetriou, 2000). The "strategy" category includes a great deal of knowledge that can be acquired concerning what strategies are likely to be effective in achieving what goals and in what sort of cognitive undertakings.

Whereas Flavell uses taxonomy with the three categories (person, task, strategy) to define metacognitive knowledge, Brown (1987) has categorized metacognitive knowledge based on a person's awareness of his/her metacognitive knowledge: declarative, procedural and conditional. Declarative knowledge is prepositional knowledge which refers to "knowing what", procedural knowledge refers to "knowing how" and conditional knowledge refers to "knowing why and when". For example in mathematics education the knowledge that "I am not good in remembering mathematical rules" is a declarative knowledge, the knowledge that "I can remember information easier if I connect them with everyday regular experiences" is a procedural knowledge and knowing that "I reduce the big numbers of a problem in order to manipulate them better", is a conditional knowledge.

The second dimension of metacognition, self-regulation refers to the processes that coordinate cognition. It is the ability to use metacognitive knowledge strategically to achieve cognitive goals; especially in cases that someone needs to overcome cognitive obstacles. It has become clear that one of the most important issues in self regulated learning is the students´ ability to select, combine and coordinate strategies in an affective way (Boekaerts, 1999). Successful learners are able to swiftly transfer the knowledge and strategies acquired in one situation to new situations, modifying and extending these strategies on the way. Self- regulated learners in school age are able to manage and monitor their own processes of knowledge and skill acquisition (DeCorte, Verschaffel & Op´t Eynde, 2000). Self-regulatory behavior in mathematics includes clarifying problem goals, understanding concepts, applying knowledge to each goal and monitoring progress toward a solution.

The relationship between cognitive and metacognitive processes is one of the issues elaborated by Demetriou and his colleagues (Demetriou & Kazi, 2001; Demetriou, et al., 2002). Actually the present study was motivated by the intergrated model proposed by Demetriou according to which, the mind includes three fundamental levels of organization. The structure of the mind, referring to the cognitive and metacognitive system is presented at the next section.

2. The Structure of the mind: processing efficiency, working memory and metacognition

The human mind can be described as a three level hierarchical system involving domain-general and domain-specific processes and functions (Demetriou & Kazi, 2001; Demetriou, 2004). Speed of processing, inhibition and control, and working memory are the basic dimensions that define the condition of this system.

According to this model, the mind includes two levels of knowing, one oriented to the environment and another oriented to the self. That is, the first level includes representational and understanding processes and functions that specialize in the representation and processing of information coming from the environment. The second level includes functions and processes oriented to monitoring, representing, and regulating the environment-oriented functions and processes. Thus, the input to this level is information arising from the functioning of the environment-oriented systems under the current processing constraints (for example, sensations, feelings, and conceptions caused by mental activity). Optimum performance at any time depends on the interaction between the two levels, because efficient problem solving or decision making requires the application of environment oriented functions and processes, under the guidance of representations held about them at the level of self-oriented processes.

Two of the main cognitive processes that the present study investigates are information processing and working memory. The processing system is defined in terms of three main parameters: speed of processing, control of processing and working memory. The first parameter is the maximum speed at which a given mental act may be efficiently executed; it refers to the time needed by the system to record and give meaning to information and execute an operation. Control of processing determines the system's efficiency in selecting the appropriate mental action. The more demanding a task is, the more processing resources, monitoring, and regulation it requires. Finally, working memory refers to the quantity of processes, which enable a person to hold information until the current problem is solved (Demetriou & Kazi, 2001). A common measure of working memory is the maximum amount of information and mental acts that the mind can efficiently activate simultaneously.

The present study outlines this architecture with an emphasis on the interdependent relations of the system. This architecture has similarities and differences with models proposed by psychometric theories, which are presented below. The emphasis is on the impact of processing efficiency and working memory ability on the development of mathematical performance and metacognition.

2.1 The development of cognitive and metacognitive abilities

The neo-Piagetian perspective, as Demetriou´ s theory, explains the cognitive development in terms of information processing. The limits in working memory capacity impose constraints on cognitive processes, and vary with age. New Piagetian theorists consider the development of working memory to be a causal factor of cognitive growth across domains (Kemps, Rammelaere, & Desmet, 2000). The core of neo-Piagetian research is to explore whether the development increase in working memory can account for cognitive development at large.

There is evidence that processing speed changes uniformly with age, in an exponential fashion, across a wide variety of different types of information and task complexities. That is, change on speed of processing is fast at the beginning (i.e., from early to middle childhood) and it decelerates systematically (from early adolescence onwards) until it attains its maximum in early adulthood (Demetriou et al. 2002; Hale and Fry, 2000; Kail, 1991). This pattern of change reflects the fact, that, with age, the time taken by the brain to complete an operation becomes smaller due to improvements in the interconnectivity of the neural circuits in the brain and the improvements in the myelinization of neuronal axons that insulate the communication between neurons. As a result, the representation and manipulation of information in the brain becomes faster and more efficient (Case, 1992; Thatcher, 1992).

Concerning the working memory there is general agreement that the capacity of all components of working memory (i.e., executive processes, phonological, and visual storage) do increase systematically with age. Additionally, there seems to be an inverse trade-off between the central executive and the storage buffers, so that the higher the involvement of executive processes the less is the manifest capacity of the modality-specific buffers. This is so because the executive operations themselves consume part of the available processing recourses. However, with age, executive operations and information are chunked into integrated units. As a result, with development, the person can store increasingly more complex units of information (Case, 1985).

Concerning the metacognitive abilities Kail´ s research (1991) indicated that even preschoolers are capable of reflecting on their own prior knowledge. By the time young children begin to express and recognize them as enduring entities, they also begin to show major advances in their understanding of others (Rochat, 2003). Actually by 4-5 years, according to Schneider and Sodian (1998), children begin to be capable of holding multiple representations of themselves and others. By the age of about 4 years, children understand the relation between beliefs and knowing, while between the ages of 4-7 years children move to a more sophisticated understanding of the role of inferential processes in knowledge acquisition (Schneider & Sodian, 1998). However, it is important the investigation of the development of specific metacognitive dimensions, such as self-representation, in relation to cognitive abilities in childhood ages.

As we have already mentioned, the purpose of the present study was to investigate the interrelations among the cognitive processes of information processing and working memory with mathematical performance and the inner metacognitive process from a developmental perspective, depending on the model of Demetriou and his colleagues. The architecture of the mind postulated by this model bears similarities and differences to architecture postulated by others models, for example the hierarchical conception of the human intelligence, proposed by Custafsson (1988). Moreover, many of the functions proposed are common with the abilities described by psychometric theories (Case, Demetriou, Platsidou & Kazi, 2001). However, the present study aimed to go beyond general cognitive structure and include measures of both actual cognitive performance and metacognition, at the specific domain of mathematics and not generally at the whole performance. Even though, children's early understanding of themselves has been intensively investigated in the last decades, there is a lack of studies investigated at the same time cognitive abilities and metacognition, in respect to specific domains, such as mathematics. Particularly in mathematics education, research should be concentrated on the impact of cognitive factors on the development of the metacognition at the specific domain, and consequently on the respective performance. A reliable model depicting the development of those cognitive and metacognitive abilities could be useful in two ways: On the theoretical level it will contribute to deeper understanding of this important interconnection and on the practical side it may be useful in developing teaching programs for the improvement of young pupils´ metacognition in mathematics.

3. Method

In the present study we developed and used a self-reported inventory measuring metacognition and an inventory measuring mathematical performance. The exact procedure, which was used for the measurement of processing efficiency, working memory, mathematical performance and metacognition, is presented below.

3.1Participants

Data were collected from 126 children, in grades three through five (about 8 to 11 years old), from six different urban elementary schools. Specifically, 37 (19 girls, 18 boys) were 3rd graders, 40 (18 girls, 22 boys) were 4th graders and 49 (24 girls, 25 boys) were 5th graders. The mean age of the overall sample was 9.5, with students ranging in age from 7.9 to 11.4 years, at the first time of testing. The mean age of the 3rd graders was 8.4, with students ranging in age from 7.9 to 8.8. The mean age of the 4th graders was 9.5 with students ranging in age from 8.9 to 9.9 and finally the mean age of the 5th graders was 10.7 years, with students ranging in age from 10 to 11.2.

3.2Materials

Apart from self-reporting inventory for the measurement of metacognitive performance and mathematical performance, individual meetings were arranged with each one of the subjects for the measurement of processing efficiency and working memory.

3.3The inventory for the measurement of the metacognitive performance

The inventory for the measurement of the metacognitive performance was comprised of 30 Likert type items, of five points (1=never, 2=seldom, 3=sometimes, 4=often, 5=always), reflecting pupils perceived behaviour during in-class problem solving. A specimen item is: “when I encounter a difficulty that confuses me in my attempt to solve a problem I try again”. The responses provide an image of pupils´ self-representation, which refers to how they perceive themselves in regard to a given mathematical problem. The 30 items are presented at the Appendix, as a part of a table presenting the factor loadings of them (Table 1). The reliability of the whole inventory was very high. Specifically, the Cronbach´ s α was 0.86.

3.4Mathematical performance tasks

The individual’s mathematical ability was measured through four numerical tasks, four analogical, four verbal and four matrices for the measurement of spatial ability taken from the Standard Progressive Matrices. All mathematical tasks were used in previous studies (Demetriou et al., 2002). The reliability of the cognitive mathematical tasks was high. Specifically, the Cronbach´ s α was 0.87. All items in the mathematical performance inventory were scored on a pass-fail basis (0 and 1).

3.5Stroop-like tasks

The pupils' information processing efficiency was measured using a series of stroop-like tasks devised to measure speed and control of processing, under three different symbol systems: numerical, verbal, and imaginal. To measure, for example, verbal speed of processing, participants were asked to read at the computer a number of words, denoting a colour written in the same ink-colour (for example the word green written in green) and they had to type the letter G at the keyboard, indicating the written word or the colour of the word. At the half of the items the instructions were to type G (for green), Y (for yellow) and R (for red) in order to indicate, as quickly as possible, the colour of the word and at the other half of the items the instruction were to type the respective buttons in order to indicate the written word. For verbal control of processing, the subjects were asked to recognize the ink colour of words denoting a colour different than the ink (for example the word green written in red). To measure the two dimensions of numerical processing, several number digits were composed of small digits. This task involved the numbers 4, 7 and 9. In the compatible condition the large digit was composed of the same digits, while in the incompatible condition the large digit was composed of one of the other digits. Pupils had to type at half of the items the small digit and at the rest the big digit, according to the given instructions. Reaction times to all three types of the compatible conditions (verbal, numerical, imaginal) were taken to indicate speed of processing, while reaction times to the incompatible conditions were considered indicative of the person's efficient control of processing. The tasks addressed to the imaginal system were similar to those used for the numerical system and comprised three geometrical figures: circle, triangle, square. The buttons on the keyboard, they had to use, were "S" for square, "C" for circle and "F" for triangle. Two examples of the given tasks are presented at Figure 1. The computer measured reactions times automatically.

The reliability of processing efficiency tasks was very high. Specifically, the Cronbach´ s α was 0.91.

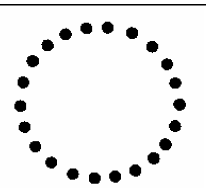	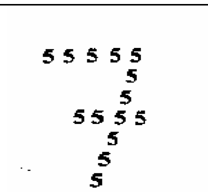
An imaginal compatible stimuli	A numerical incompatible stimuli

Figure 1: Examples of stroop like tasks

3.6 Working-memory tasks

To measure working memory, we asked pupils to recall a number of words, sets of numbers, and images. The verbal task, for example, combined six levels of difficulty, each of which was tested by two different trials. The difficulty level was defined in terms of the number of words in the task, which ranged from two to seven concrete nouns. The numerical tasks were structurally identical to the verbal task. Specifically in the easy trial, only decade numbers were involved, while in the difficult trial the two digits of the numbers were different. Both words and numbers presented to children as verbal stimuli. In the imaginal task, the stimuli were presented visually at the computer. The participants were shown a card on which a number (2-7) of geometrical figures were shown and they were asked to choose from four choices the card, which had the same figures, at the same relative position with the first one (Fig. 2).

The reliability of working memory tasks was very high. Specifically, the Cronbach´ s α was 0.83.

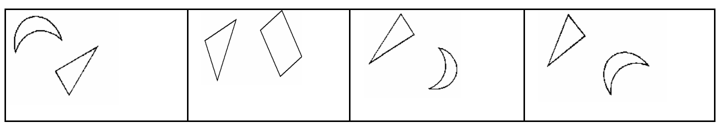

Figure 2: An example of working memory task

3.7 Procedure

To specify the nature of change in cognitive abilities in mathematics in relation to metacognition and the possible interrelations in the patterns of change in these aspects, a series of three repeated waves of measurements were taken, with a break of 3-4 months between successive measurements. The same materials were used at each wave of measurement. Each participant was tested individually on processing efficiency and working memory tasks. Testing took place in a quiet room, provided by the schools for the purpose of the experiment. The breaks of 3-4 months for the consecutive measurements were necessary in order to investigate the developmental changes of the specific variables. The investigation of the impact of development factors on specific abilities by repeated measurements with longer breaks was impossible, because of practical difficulties, which would not permit us to have the same sample of pupils.

3.8 The stoop-like tasks

The experimenter introduced the three tasks (numerical, verbal, imaginal) to the child, through the computer, one by one, using first several demonstration cards and then several practice cards to familiarize the child with the tasks, mainly in order to learn the buttons they had to use from the keyboard. For practical reasons, the presentation order of items within the symbol systems was the same across subjects.

3.9 The memory tasks

For verbal and numerical tasks, participants were instructed to recall the words or the numbers in the order of presentation as soon as the experimenter finished stating a series. The presentation order of difficulty levels was the same across participants, going from easy to difficult.

Administration of a task stopped if the participant failed to recall errorlessly the two trials involved in a level. In the spatial task, the participant was instructed to carefully choose the card with the figures in exactly the same position and orientation as the initial one.

3.10 The mathematical tasks and the inventory of metacognitive performance

The mathematical tasks and the inventory about metacognition were individually presented in a paper and pencil form and were individually administered. The experimenter explained each task and was available to answer questions as needed.

4. Results

4.1 The structure of the cognitive and metacognitive processes

The collected data of the inventory about metacognitive abilities were first subjected to exploratory factor analysis in order to examine whether the factors that guided the construction of the inventory were presented in the participants´ responses. This analysis resulted in 10 factors with eigenvalues greater than 1, explaining 64.74% of the total variance (the factor loadings and the content of the items are shown in Table 1). After a content analysis of the ten factors, according to the results of the exploratory factor analysis, these factors were classified in the following four groups: "general self-image" (two factors), "strategies" (four factors), "motivation" (two factors), and "self-regulation" (two factors). The means of those four groups of factors were subsequently used in order to avoid a big number of variables at the structural equation modeling. It is important to note that reducing a large number of raw scores to a limited number of representative scores is an approach suggested by proponents of structural equation modeling (Gustafsson, 1988). The items that constructed the two factors about "self-image" referred to the beliefs and self-efficacy that pupils had about their abilities, in general, and while encountering specific situations, in particular. "Self-regulation" in mathematics, the other two factors of the analysis, included clarifying problem goals, understanding concepts, applying knowledge to each goal to develop a solution strategy and monitoring progress toward a solution. The factors of "strategies" consisted of items concerning the strategies pupils used in order to solve problems and in order to overcome cognitive obstacles. Finally, the factors of "motivation" consisted of pupils' beliefs about the impact of their effort and their will on their performance and the impact of their parents and teachers.

Thus the first and second order factor structure of the instruments was investigated to determine whether the general levels of the architecture of the mind, that is speed of processing, control of processing, working memory and self-awareness system, explain the variability in the different means scores. Confirmatory factor analysis model designed to test the multidimensionality of the materials were used in order to examine their construct validity. It was important to investigate the degree of the similarity of the models, which could be constructed for the repeated measurements. Structural equation modelling was used to test the hypothesis on the existence of seven first order factors, two second order factors and a third order factor, in all cases. The seven first order factors were processing efficiency, working memory, cognitive performance at mathematics, self-image, self-regulation, strategies and motivation. The a priori model hypothesized that the variables of all the measurements would be explained by those factors and each item would have a nonzero loading on the factor it was supposed to measure. Analysis was conducted using the EQS program (Bentler, 1995) and maximum likehood estimation procedures. Multiple criteria were used in the assessment of the model fit. The model was tested under the constraint that the error variances of some pair of scores associated with the same factor would have to be equal. This was an indication of the LMTEST, in order to arrive at an elaborated model in which the goodness of fit-index would be good in relation to typical standards (CFI>0.9, x^2/df<2, RMSEA<0.05). This model was tested separately on the performance attained at each testing wave.

There were six measures representing the two dimensions (speed and control of processing) of processing efficiency tasks. That is, the three means representing performance on the three sets of stoop-like compatible tasks addressed to speed of processing and the three means representing performance on the three sets of stoop-like incompatible tasks addressed to control of processing through the verbal, the numerical and the imaginal symbol systems. Additionally there were six mean measures representing the easy and difficult tasks of numerical, verbal and imaginal working memory tasks. Finally there were four mean measures representing the performance of individuals on numerical, verbal and analogical mathematical tasks and matrices.

A series of models were tested separately for each one of the measurement waves. Specifically, a one-factor model was first tested. The first of the models that were tested, involved only ten first order uncorrelated factors. Kline (1998) argues, "even when the theory is precise about the number of factors of a model, the researcher should determine whether the fit of a simpler, one-factor model is comparable" (p.212). The fit of this model was very poor in all cases, as it was expected:

First testing: X^2=1013.412, df=275, X^2/df=3.68, p<0.001, CFI=0.015, RMSEA=0.160
Second testing: X^2=690.571, df=251, X^2/df=2.75, p<0.001, CFI=0.370, RMSEA=0.124
Third testing: X^2=540.141, df=229, X^2/df=2.35, p<0.001, CFI=0.477, RMSEA=0.107

The second model tested involved seven first order factors, that is processing efficiency, working memory, mathematical performance, self-image, self-regulation, strategies and motivation. Thus this model tests the assumption that each of the dimensions represented by the tasks is fully autonomous of each other. The fit of this model was also very poor in all cases:

First testing: X^2=978.402, df=275, X^2/df=3.556, p<0.001, CFI=0.115, RMSEA=0.171
Second testing: X^2=710.603, df=251, X^2/df=2.831, p<0.001, CFI=0.382, RMSEA=0.129
Third testing: X^2=552.136, df=229, X^2/df=2.411, p<0.001, CFI=0.493, RMSEA=0.118

The third model involved the three first order factors (processing efficiency, working memory and cognitive performance in mathematics) regressed on a second order factor and the four first order factors (self-image, self-regulation, strategies and motivation) regressed on a different second order factor. This model was found to fit well.

First testing: X^2=295, df=268, X^2/df=1.103, p<0.001, CFI=0.904, RMSEA=0.025
Second testing: X^2=299.481, df=244, X^2/df=1.227, p<0.001, CFI=0.927, RMSEA=0.030
Third testing: X^2=257.255, df=222, X^2/df=1.158, p<0.001, CFI=0.898, RMSEA=0.032

Finally the organization of cognitive and metacognitive processes and abilities were tested using the model shown in Figure 3. It was a three level model, which was consistent with the theory. It involved three types of factors. The seven first order factors were: processing efficiency, working memory, cognition, general self-image, self-regulation, strategies and motivation. Those factors were regressed on two second order factors: the general cognition and the general self-representation. Those second order factors of cognitive and metacognitive processes were regressed on a third order factor that concerned the cognitive and metacognitive abilities in mathematics.

First testing: X^2=279.949, df=262, X^2/df=1.068, p=0.213, CFI=0.974, RMSEA=0.026
Second testing: X^2=265.413, df=236, X^2/df=1.124, p=0.091, CFI=0.956, RMSEA=0.034
Third testing: X^2=224.374, df=195, X^2/df=1.150, p=0.073, CFI=0.952, RMSEA=0.036

The parameter estimates of this final model for the three waves are shown in Figure 3. The fit of the model was very good and the values of the estimates were high in all cases. It is clear therefore that the three-level architecture accurately captures the data.

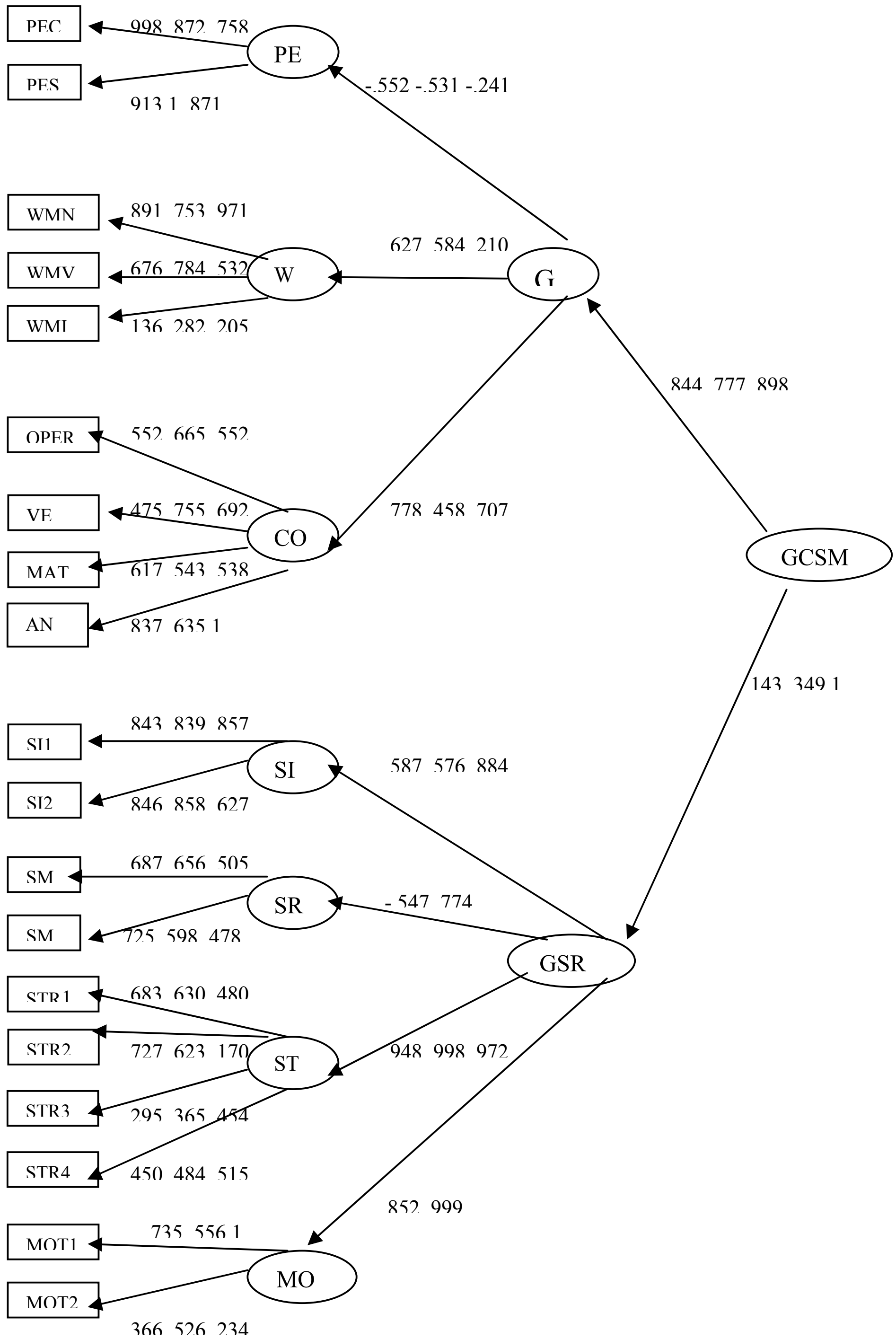

Figure 3: The third level model of metacognitive and cognitive abilities in mathematics in three testing waves[2]

[2] PE = Processing Efficiency, PEC= Control of Processing, PES= Speed of Processing, WM = Working Memory, WMN= Numerical, WMV= Verbal , WMI= Imaginal, CO= Cognitive performance in mathematics, OPER= Operations, VE= Verbal, MAT= Matrices, AN= Analogies, SI = Self Image, SR = Self-regulation, STR= Strategies,

4.2 The development of cognitive and metacognitive abilities

In order to specify the dynamic relations between mathematical performance and metacognition with processing efficiency and working memory, during the period of the study, dynamic modelling was used. The dynamic model explored possible relations among cognitive variables (processing efficiency, working memory, and cognitive performance in mathematics) and metacognitive variables (self-image and self-regulation) across the three waves of measurement. The variables of strategies and motivation excluded from the last analysis in order to avoid testing a complicated model with too many variables and consequently many limitations with the statistical analysis. We believed that self-image and self- regulation had a stronger relationship with the general self-representation than the use of strategies and motivation. Self-image about personal strengths and limitations, in comparison to the abilities of others, is a part of the general self-representation. While self-regulation is one of the two basic dimensions of metacognitive ability and it is too important in order to overcome obstacles encountering while solving a mathematical problem.

The dynamic model explored relations among cognitive and metacognitive variables across the three waves of measurement. The main hypothesis was that all the variables at the second measurement were affected by the respective variables at the first measurement and the variables at the third measurement were affected by the respective variables at the first and the second measurement. Furthermore, the second hypothesis was that significant relations would connect the different cognitive and metacognitive variables at each wave of the measurements.

The initial fit of the model tested, without any correlations among the five variables (processing efficiency, working memory, cognitive performance in mathematics, self-image, self-regulation) in each wave of measurement, was very poor (X^2=999.359, df=410, X^2/df=2.42, p<0.001, CFI=0.581, RMSEA=0.114). It improved, however, dramatically after the above two hypotheses were tested (X^2=482.319, df=376, X^2/df=1.28, p=0.001, CFI=0.924, RMSEA=0.051), indicating the impact of the first measurement on the respective abilities at the second and the third measurements and the connection of the different cognitive and metacognitive abilities at each wave of the measurements. After a few error variances were allowed to correlate, according to the indications of the LMTEST, the fit of the model was excellent (X^2=434.964, df=373, X^2/df=1.16, p=0.01, CFI=0.956, RMSEA=0.039). The parameter estimates of this model are shown in Figure 4.

The results of the above dynamic model underline the predominance of the processing efficiency and the working memory for the structure of the cognitive mathematical performance and

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MOT= Motivation, GSR= General Self Representation, GC = General Cognitive Abilities, GCSM= General Cognitive and Self-representation Abilities in Mathematics

The three numbers indicated the loadings of the variables and factors at the three consecutive measurements, respectively.

Detailed tables with Means, Standard Deviations and Correlations between the Variables Used in Structural Modeling can be obtained from the author.

individuals' metacognitive performance. The cognitive performance in mathematics at the third measurement (COG3) was affected significantly by the initial condition of the processing (PE1) efficiency (0.397) and the condition of working memory (WM2) at the second measurement (0.495). It was important that the effect of the initial mathematical (COG1) performance (.226) was lower than the effect of the initial processing efficiency (PE1) and the previous working memory ability (WM2). Individuals΄ differences on the two cognitive processes remained the same after one year and specify the differences at the cognitive performance and the self-image. In particular, these results indicated that individuals with high processing efficiency and high working memory ability had high self-regulation ability and a positive self-image. It is very important that there were no statistically significant correlations between self-image and self-regulation with the cognitive performance in mathematics, at the first and second measurement. This indicated that the individuals΄ self-image, except the third measurement, did not depend on their mathematical performance. This was a non-expectable result that underlined the important role of working memory on the development of the cognitive system and the impact of the most recent experiences on the structure of the self-image.

The model parameters (Figure 4) show that there was a general pattern of individuals΄ differences at the first measurement that persisted at the second and the third measurement in the case of working memory. This is evidenced from the continuing significant loadings of each variable at different measurements. Specifically, the loading of the working memory ability at the first measurement (WM1) on the working memory ability at the second measurement (WM2) was 0.814. Similarly, the loading of the working memory ability at the first measurement, and the loading of the same variable at the second measurement on the working memory ability at the third measurement (WM3) were 0.767, and 0.349, respectively. The behaviour was not the same in the case of the other variables. The difference remained at the second measurement, but changed at the third one.

A notable finding from the specific dynamic model was the predominant role played by the processing efficiency, affecting significantly all the others cognitive and metacognitive variables at the first measurement. The statistically significant loading of processing efficiency (PE1) on working memory (WM1) was -0.337, on mathematical performance (COG1) was -0.206, on self-image (SI1) was -0.198 and on self-regulation (SR1) was -0.262. At the same time, the predominant role of processing efficiency on the whole system was underlined by the result that the loading of processing efficiency at the first measurement on cognitive mathematical performance at the third measurement was significant (-0.397). The loading of working memory at the second measurement (WM2) on the mathematical performance at the third measurement (COG3) was significant as well (0.226). Consequently the mathematical performance depended on the previous processing efficiency and the working memory.

The performance of self-image at the third measurement (SI3) was affected by the initial condition (WM1) of the working memory (0.239) and the mathematical performance (COG3) at the third measurement (0.530). This is an important indication of the factors that affect individuals΄ self-image in mathematics. Actually the impact of the mathematical performance at the same measurement was expectable, because of the recent experiences. Nevertheless the impact of the initial condition of the working memory ability indicated the predominant role of cognitive processes and abilities on the self-image.

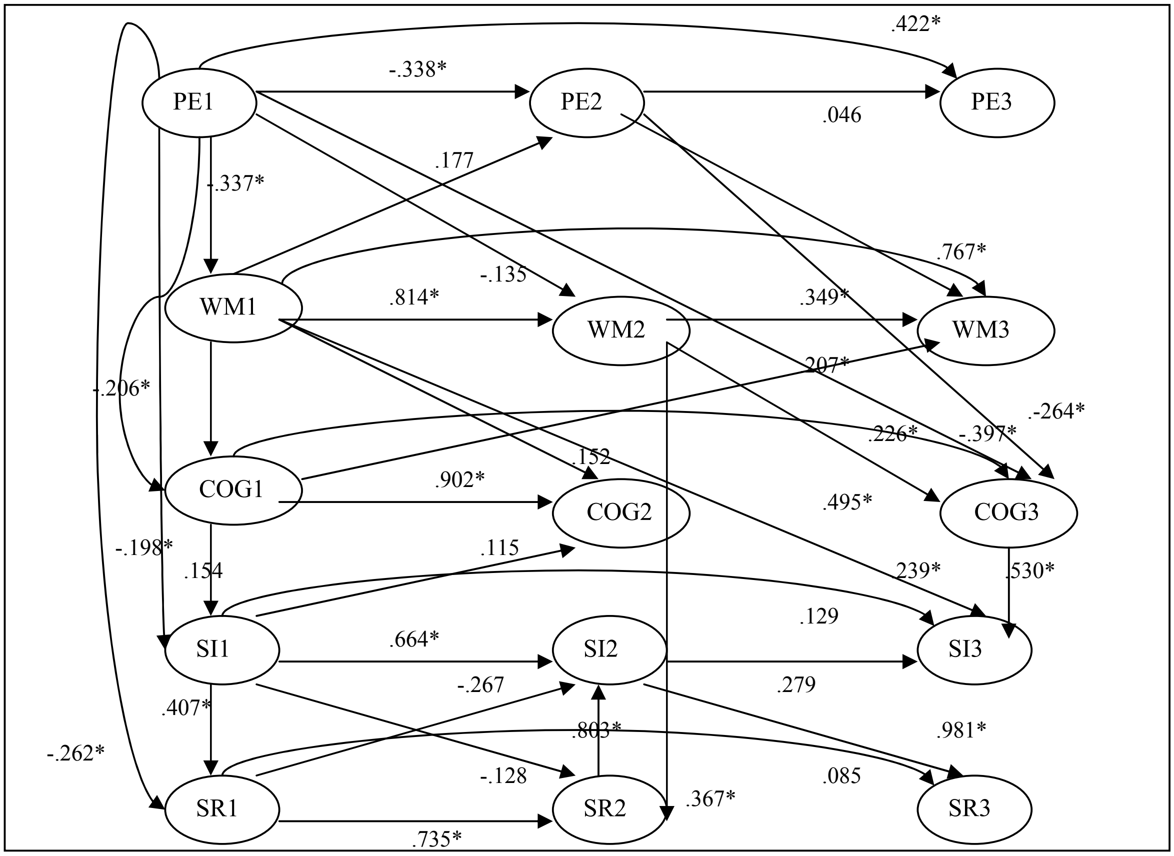

Figure 4: The dynamic model of cognitive and metacognitive abilities through the three measurements[3]

We next used growth modeling to explore the nature of change in cognitive and metacognitive abilities in mathematics, and the possible interrelations in the patterns of change in these variables. This model was estimated with the MPLUS packet (Muthen & Muthen, 2001). The basic latent growth model was composed of two latent factors: The first one represented the initial status – the intercept, and the second one was the latent growth rate – the slope, and was defined by fixing it to 0, 1, and 2. Figure 5 illustrates the general model that was tested. The following twelve manifest variables were used in this model: three for processing efficiency, three for working memory, three for cognitive memory, three for cognitive ability in mathematics and three for self-image. Each variable was a composite measure of the performance attained on the tasks at each of the three testing waves. Thus, processing efficiency

[3] PE= Processing Efficiency, WM= Working Memory, COG= Cognitive Abilities in Mathematics, SI= Self Image, SR= Self-regulation
* Significance at the .05 level
1= first measurement 2= second measurement 3 =third measurement

was the mean of performance attained on the processing speed and control of processing tasks; working memory was the mean of performance attained on numerical, verbal and imaginal memory tasks; cognitive ability in mathematics attained on the performance on numerical, verbal, analogical mathematics tasks and matrices. Self-image was the mean of the two factors, which were the results of the exploratory factor analysis for self-image.

At the initial run of the model, the slope variable was fixed to have a relation of 0, 1, and 2 with all manifest variables at the first, second and third testing waves, respectively. This constraint expresses the modelling assumption that change is a linear function of time. The slope of the processing efficiency was the only fixed variable; it was assigned the relation of -2, -1, 0, because of the predictable reduction of reaction time. For the best fitting of the model the intercepts and the slopes were changed, as shown in Figure 5, in order to justify the differences of means, without changing the linearity of the model (e.g., the slope variable for the cognitive performance was fixed to have a relation of 0, 0.38, 0.90). The overall model fit statistics were X^2=64.60, df= 41, X^2/df=1.575, p=0.001, CFI=0.934, RMSEA=0.06, suggesting an excellent fit of the model to the data. The parameter estimates of the model are presented in Figure 5.

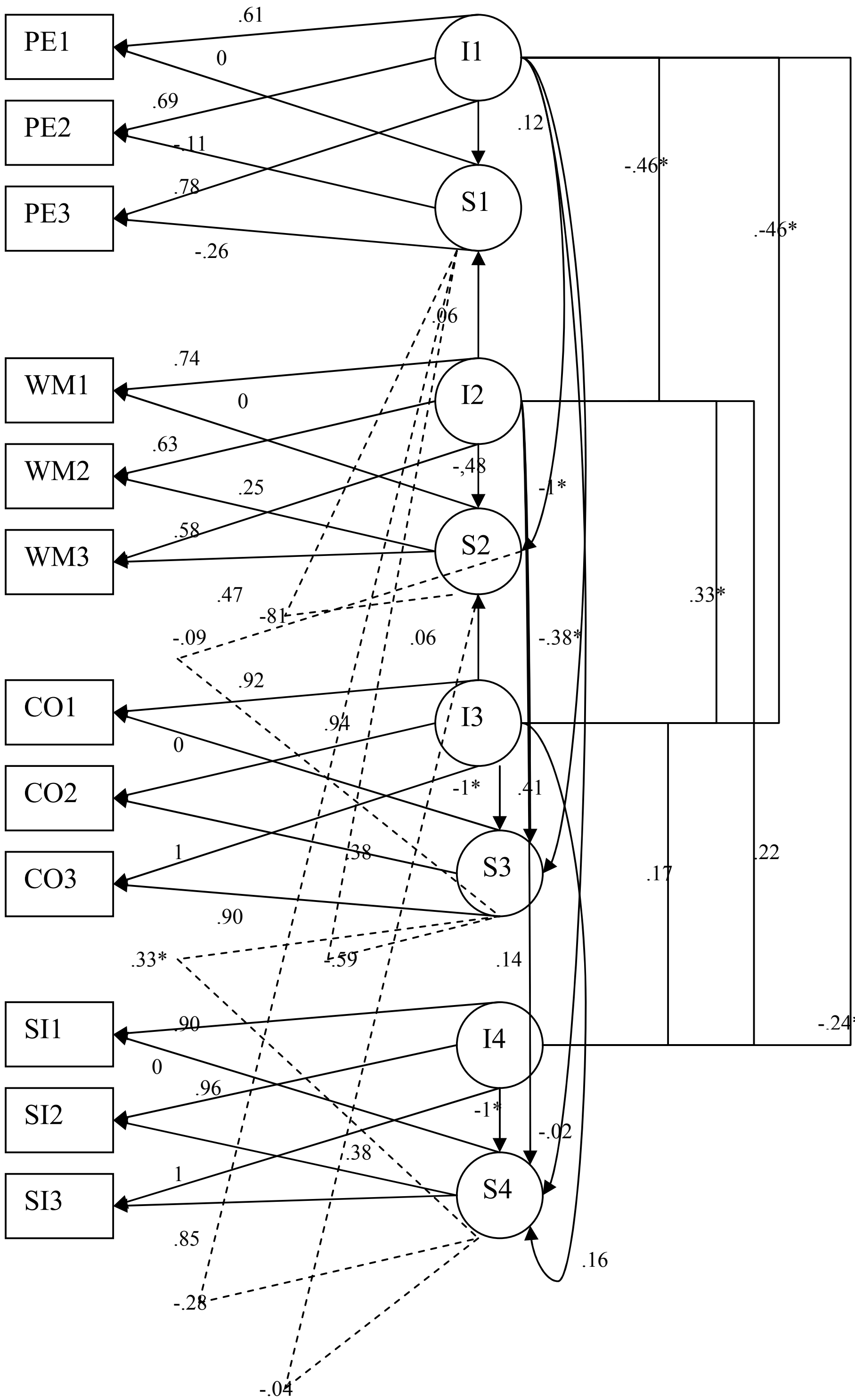

Figure 5: The best fitting growth model for processing efficiency, working memory, cognitive mathematical performance and self-image across the three testing waves[4].

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[4] PE = Processing Efficiency, WM = Working Memory, CO = Cognitive performance in mathematics, SI = Self Image, I = Intercept, S = Slope

The correlation among the intercepts and between the intercept and the slope were very important. The regression of the processing-efficiency intercept on the working memory intercept and the regression of the working memory intercept on the cognitive abilities intercept were not significant. The correlations between the intercept of processing efficiency with the slope of working memory and the intercept of processing efficiency with the slope of cognitive ability were significant. Both of them were negative (-0.46, -0.38, respectively) meaning that individuals who had higher mean value of the initial scores on the time of processing efficiency (meaning slower speed) had a weaker rate of increase. Quite notable is the finding that the initial condition of self-image depends on the corresponding processing efficiency (-0.24), while its growth depends on the growth of mathematical performance (0.33). The positive correlation among the growth of the two variables indicates that individuals who improved on mathematical performance had improved on self-image as well.

The pattern of findings presented, suggest several important conclusions. First, that there are significant individual differences in the pupils' attainment in cognitive and metacognitive variables. The significant correlations among the intercepts indicate that there are strong interrelations among the initial conditions of the three cognitive aspects of the mind and the metacognitive aspect. However, growth over time affects the development of each function of the mind differently. The fact that the slope of cognitive ability and working memory depended on the initial condition of the processing efficiency indicated that processing efficiency had a coordinator role on the development of other cognitive and metacognitive processes.

The existence of significant intercept correlations among different abilities (processing efficiency with working memory and processing efficiency with cognitive performance in mathematics) suggest that growth in each of the abilities was affected by the state of the others, especially the state of processing efficiency at a given point of time. On the other hand, the lack of intercept – slope relations between self-image and cognitive abilities suggests that growth of self-image was not affected by the state of the processing efficiency or working memory at a given point in time. The relation of the slope of self-image with the slope of cognitive abilities indicates that the advancement on self-image depended on the advancement of mathematical performance.

5. Discussion

The findings of this study lead to some potentially important conclusions about the development of cognitive and metacognitive processes. Although complicated figures are presented at the above section, few are the results that should be underlined regarding the interrelations metacognition, cognitive processes and mathematical performance. Firstly, there was stability on the models which were constructed for the repeated measurements, indicating the stability of the structure of the specific variables the study investigated. Secondly, results indicated that the development of each of the cognitive abilities and dimensions of metacognition was affected by the state of the others. Particularly, processing efficiency had a basic impact on the growth of mathematical performance and on working memory. The mathematical performance depended on the previous working memory ability, as well. Finally, the self-image, as a significant part of self-representation, depended mainly on the previous working memory ability and on the recently mathematical performance.

The human mind is much more complex than simply cognitive abilities and processes and their presentations (Demetriou & Kazi, 2001). Metacognition is constrained by the processing potentials of the mind. The existence of significant correlations among different cognitive abilities, especially between processing efficiency with working memory and cognitive performance in mathematics suggest that growth in each of the abilities is affected by the state of the other variables, especially the state of processing efficiency at a given point of time. From the analysis of the dynamic model, it is quite clear that the processing efficiency has a coordinator role on the cognitive system and the individual΄ s metacognitive performanfce, even from the first measurement. This result was observable from the fact that processing efficiency is strongly associated with all the other factors, and actually affects them significantly.

The lack of relations between self-image and cognitive abilities suggests that growth of metacognitive performance is not directly affected by the state of the processing efficiency or working memory, at a given point of time. It is mainly affected by the initial condition of those abilities. Individuals΄ self-image depended mainly on previous working memory ability and partially on the recent mathematical performance. It is very important the effect of mathematical performance on the self- image at the final measurement. It seems that mathematical performance is the only cognitive ability, for which individuals have direct consequences which are expressed by remarks, awards and most often rewards by significant others i.e., teachers and parents. This is in line with Reder (1996) conclusions that consciousness is a necessary condition of a more precise self-representation.

Demetriou et al. (2002) suggest that both the working memory and the processing efficiency are associated with individual development differences on thinking. A change at the metacognitive system influences the functioning of the cognitive system and vice-versa. The results of the present study indicated that changes on thinking and metacognitive performance might be associated with processing efficiency and working memory, even at the years of the primary education, at the specific domain of mathematics. Additionally the present study found that change on processing efficiency or working memory may be necessary, but not sufficient for changes on functions residing at other levels of the mental architecture.

The present study has provided evidence about the relations and interconnections among cognitive and metacognitive processes with respect to mathematical performance. It is too important the predominant role of processing efficiency and working memory ability on mathematical performance and metacognition. Further investigation could lead to intervention programs for the improvement of self-representation in mathematics. Future studies could investigate whether changes on cognitive performance, especially on cognitive processes, such as processing efficiency and working memory capacity, tend to follow changes on metacognitive knowledge, self-evaluation, self-regulation and self-representation. In the area of mathematics, a number of important questions remain unanswered about metacognition. Are people aware of cognitive processes when they do mathematics, even in early childhood? Are they accurate in their self-representations of strengths and weaknesses in mathematics? Much more research is needed to study the different aspects of metacognition in a more systematic, detailed way. It should continue on the possible developmental changes in the interrelations among specific cognitive processes and metacognitive processes.

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Questionnaire items	Factors								
	SI1	SI2	SR1	SR2	STR1	STR2	STR3	STR4	MOT1
I examine my own performance while I am studying a new subject.	.759								
When I read a problem I know whether I can solve it.	.599								
After I finish my work I know how well I performed on it.	.696								
I know ways to remember knowledge I have learned in Mathematics.		.823							
I understand a problem better if I write down its data.		.640							
When I cannot solve a problem, I know the factors of the difficulty.		.605							
I know how well I have understood a subject I have studied.		.539							
I define specific goals before my attempt to learn something.			.736						
After I finish my work I wonder whether there was an easier way to do it.			.680						
When I encounter a difficulty on problem solving I reread the problem.			.639						
When I encounter a difficulty that confuses me in my attempt to solve a problem I try to resolve it.			.758						
I can learn more about a subject on which I have previous knowledge.				.650					
I can learn more about a subject on which I have a special interest.				.763					
When I am solving a problem I wonder whether I answer its major question.				.580					
I try to use ways of studying that had been proved to be successful.					.549				
For the better understanding of a subject I use my own examples.					.782				
In order to solve a problem I try to remember the solution of similar problems.					.557				
I understand something better if I use pictures or diagrams.						.648			
I concentrate my attention on the data of a problem.						.634			
When I try to solve a problem I pose questions to myself in order to concentrate my attention on it.						.553			
After I finish my work I wonder whether I have learned new important things.							.754		
After I finish my work I repeat the most important points in order to be sure I have learned them.							.633		
Before I present the final solution of a problem I try to find some other solutions as well.							.658		
When I do not understand something I ask for the help of others.								.671	
While I am solving a problem I try to realize which its aspects that I cannot understand are.								.730	
When I encounter a difficulty in problem solving I am looking for teacher's help.								.505	
My performance depends on my will.									.778
My performance depends on my effort.									.599
My teacher believes that I must be a good student in mathematics.									
My parents believe that I must be a good student in mathematics.									

Table 1: Varimax Rotated Factor Matrix