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Book review: *Theories of Mathematics Education: Seeking New Frontiers*

Sriraman, B. & English, L., Eds. (2010). *Theories of Mathematics Education: Seeking New Frontiers* (Advances in Mathematics Education). Berlin/Heidelberg: Springer Science. $129. ISBN: 978-3-642-00741-5. 668 pgs

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The number of theories used in mathematics education research keeps growing, especially since what Stephen Lerman describes as the field's "social turn" (p. 103) in the 1990s. What does this proliferation of theory mean for our field? How can we synthesize findings from so many perspectives and frameworks in order to make progress?

In *Theories of Mathematics Education: Seeking New Frontiers*, a multitude of voices discuss this abundance of theory. The book contains 19 parts, most of which consist of a thematic chapter, at least one commentary on that chapter, and a preface, resulting in a grand total of 60 pieces written by 52 contributors from 13 countries. To make the endeavor even more complex, about a third of the book's chapters have been published previously, many as articles in 2005 and 2006 issues of *Zentralblatt für Didaktik der Mathematik* (ZDM), enabling each such chapter to be explicitly commented on not only by its preface and commentaries but also by many of the other chapters in the book. The purpose of this innovative and elaborate commentary structure is to put these theories and observations, many of which have already appeared in some form elsewhere, into conversation with one another. The resulting interplay of comment and interpretation feels like a boisterous town hall meeting: dynamic, diverse, and sometimes digressive. This lively discussion moves through such a wide variety of theory-related topics that the few of us left in the field who did not write part of this book will surely find in it something of interest.

The book's purpose, to which the unusual commentary structure is well-suited, is to foster a rich meta-level discussion about theory in mathematics education research (hereafter MER): what theory is for, how it should be used, refined, developed, etc. Thus many of the book's chapters present observations and theories *about* mathematics education and its frameworks. However there are also many chapters of a second kind, chapters that instead simply display particular theories being used *in* mathematics education without talking about theory at a meta-level. These chapters constitute a somewhat capricious sampling of particular theories being used in MER. Some of these particular theories do happen to illustrate the meta-level observations and pronouncements made in the book's chapters of the first kind, but this usually feels like an accident, particularly since their accompanying commentaries often do not discuss these theories at a meta-level. To be sure, these chapters are some of the most lucid and insightful in the book, but it is not always clear why they are included in this volume, especially the ones that have already been published elsewhere. To give an overall picture of the book, I will discuss a few representative examples of these two types of chapters.

Theories about mathematics education

One approach proposed for negotiating the multitude of diverging theories in MER is to work toward a grand theory to help organize and orient the field (Silver & Herbst, 2007). In Part I, Bharath Sriraman and Lyn English claim that such an enterprise is undesirable and futile, since the social and cultural settings in which mathematics teaching and learning occur are too various to allow for such sweeping abstractions. Frank Lester agrees in Part III, proposing that researchers should negotiate the “sometimes bewildering array of theoretical alternatives” (pp. 82-83) by acting as bricoleurs, handymen who use whatever tools are available to solve the problem at hand. Instead of striving for some elusive systematicity or starting with a rigid ideology, we should be proficient with a flexible array of theoretical tools.

In her commentary in Part VI, Bettina Dahl worries that such a practice leads to what she calls “uncritical complementarism” (p. 202). Rather than picking and choosing as we go, “in the absence of an overall theory, we at least need a meta-theory to help us make these informed decisions” (p. 202). This is a central issue of the book — how much meta-theory (or meta-meta-theory, etc.) is needed in order to productively guide the domain of MER?

In Part XVI, Uwe Gellert discusses a different problem with applying the bricoleur metaphor to MER. While he agrees that a researcher must select theoretical tools that are appropriate to the particular research project, he points out that a bricoleur cleverly makes do with whatever tools are around, but does not spend time either looking around for better tools or focusing on refining and improving the tools themselves. If MER is to progress as a field, we must not just solve problems or accumulate specific findings, but also continually refine and develop the theoretical tools that we have been using to interpret our data and guide our inquiry.

Richard Lesh and Bharath Sriraman agree that it is this iterative process of designing, testing, and redesigning theoretical tools and artifacts that is crucial to MER, and in Part V they claim that we should explicitly reconceptualize our field as a design science. A design science, like architecture or engineering, is geared around studying complex adaptive systems involving the interaction of subjects, systems in which it is difficult or impossible to isolate or control for variables. Lesh and Sriraman are by no means the first to advocate such an approach, and MER has included such iterative design practices for years. But the authors make headway by producing an overarching framework for explicitly employing design principles in conducting and discussing MER.

The most exciting aspect of this chapter is how it positions MER's purpose as producing successful design science rather than faulty scientifically-based research. As English notes in her preface to Part V, MER has endured many misplaced attacks, such as from the 2008 U.S. National Mathematics Advisory Panel Report, for conducting studies other than large-scale randomized controlled trials. In Part IV, Lerman expresses frustration with the repeated "attacks on educational research as an inadequate shadow of a fetishised image of scientific, psychological or medical research" (p. 108). Such narrow research designs are inappropriate for much of MER, which studies phenomena arising in complex adapting systems, and which seeks to understand why these phenomena occur rather than simply determining "what works."

Reconceptualizing MER as a design science offers a framework for furthering and defending the studies already being pursued. Lesh and Sriraman claim that it will also provide a way to address the foremost problem facing MER: lack of accumulation. By deliberately focusing on iterative design cycles and tangible products, the field will not be resigned to continually reworking the same problems in different terms. However, in his commentary, David Boote argues that plenty of design sciences have a poor track record of accumulation too. Unless its advocates can more thoroughly show just how it will better accumulate knowledge under this framework, Boote claims that MER as a design science may "become yet another educational fad" (p. 159).

In Part IV, Lerman's chapter together with Eva Jablonka and Christer Bergsten's commentary discuss how this lack of accumulation stems from the basic nature of education research, which Bernstein categorizes as a horizontal discourse with a weak grammar. Such a discourse is established "through a process of reinterpretations of the empirical more than through theoretization of objects," and thus tends to develop laterally rather than cumulatively (Moore, 2006, p. 40). While Bernstein sees such a discourse as evidence of poor theorizing, Lerman is optimistic about the plurality of theories in MER and sees the weakness of the grammar as a strength that allows for the building and integration of theories across discourses.

It is one thing to recommend the lateral integration of multiple theories; it is another to show how to do it. In Part XVI, Helga Jungwirth and Uwe Gellert each provide a concrete example of how to network a pair of theories into a theoretical framework that is more interpretively powerful and productive than either theory alone. These articles illustrate the strategies for networking theories presented in the previous part by Angelika Bikner-Ahsbahs and Susanne Prediger. Many readers would find these strategies (*comparing*, *contrasting, coordinating,* etc.) to be so broadly stated as to be scarcely useful were they not exemplified by the chapters in Part XVI. Gellert gives us a particularly lucid example of the local integration of two theories, in this case semiotics and pedagogical structuralism. He convincingly shows how this integration not only provides a richer interpretation of a given classroom event than either theory taken alone, it also produces new research questions and development of theory. This is exciting — it shows how MER can break out of Bernstein's dichotomy, in which discourses must be either vertical (knowledge builds within theories) or horizontal (new theories develop alongside existing ones). Parts XV and XVI about the networking of theories provide an optimistic, fresh, and useful stance on the multiplicity of theories in MER.

One reason these two parts on networking strategies are so successful is that the meta-level framework about how to use theories is shown through specific examples. Some meta-theoretic commentary in other parts of the book is not illustrated as tangibly, and in a few cases it is not at all clear exactly how such pronouncements could be actually acted upon by the MER community. The 'meta- upon meta-' effect is amplified by the book's commentary structure, and I admit I experienced a bit of vertigo at one point when I noticed that I was reading a commentary on an interpretation of a framework for organizing systems of theories. While such meta-level pronouncements will provoke interesting conversations, many readers will get more use out of the chapters that present specific theories of teaching and learning in actionable detail.

Theories in mathematics education

In Part XVII, Andy Hurford agrees with Lesh and Sriraman's point in Part V, that MER often focuses on behavior arising in complex adaptive systems and is problematic to study with techniques that require the control of variables. He details three frameworks of complexity theory that could serve as tools for interpreting such behavior in mathematics education: Casti's framework of complexification, Camazine et al.'s model of complex biological systems, and Holland's theory of complex adaptive systems. These theories are described in enough detail to arouse the interest of readers and point them in the direction of further work. This is one of the few chapters not accompanied by a commentary.

In Part XI, Guershon Harel describes and illustrates his DNR (Duality, Necessity, and Repeated Reasoning) framework for mathematics instruction, including the framework's premises, concepts, and instructional principles. After providing a detailed account of a classroom episode, he interprets and analyzes the episode using the theory. The analysis of the episode reveals how lucid the three instructional principles are for both guiding and interpreting classroom experiences. The necessity principle, that a student is ready to learn something only when they are experiencing an intellectual need for it, is particularly clear in the episode. DNR indeed exemplifies some of the meta-level prescriptions found in the book's earlier chapters: The framework builds upon prior theories, carefully attends to the consistency of its assumptions, provides a powerful tool for interpreting data, and has clear implications for guiding classroom practices. Mysteriously, the commentary does not discuss any of this, leaving the reader to figure out how the chapter fits in with the rest of the book.

In Part VI, John Pegg and David Tall leave aside any meta-level theorizing about when and if it is possible to make a meta-level synthesis of several large theoretic frameworks; they simply make such a synthesis. Their goal is to coordinate how learning happens locally (cognitive processes and concepts of the individual) and globally (long-term cognitive growth and development of the individual). In the SOLO (Structure of the Observed Learning Outcome) framework, first developed around 30 years ago by Collis and Pegg based upon the ideas of Piaget and Bruner, a learner's responses to a particular concept progress through unistructural, multistructural, and relational levels, forming a local learning cycle. As this cycle is repeated again and again, each time on concepts newly created from previous cycles, the learner develops through the global modes of ikonic (from 2 years old), concrete symbolic (from 6 or 7 years), and formal (from 15 or16 years) thinking. The authors then discuss how the local cycles of concept formation in two more theories (encapsulation/reification and procepts) mirror the local levels in SOLO. Finally, they discuss how Tall's three global 'worlds of mathematics' (embodied, symbolic, and formal) each favor particular local ways of forming concepts (categorization, encapsulation, and definition-deduction, respectively). It's an elaborate chapter, and I agree with Stephen Hegedus's recommendation in the preface that the interested reader should look for additional details and examples of these theories in other publications. This chapter marks some real progress, showing how a handful of cognitivist constructs all describe roughly the same things and how these frameworks can be seen as converging rather than splintering. A few instances in which this synthesis seems a bit forced are easily excused by the magnitude and optimism of the task.

In Part X, Stephen Campbell argues that the recent findings and techniques of neuroscience research are opening new avenues of investigation for MER. He describes some mathematics education research questions that are made answerable by our ability to measure things like brain activity, galvanic skin response, eye-tracking, and heart rate. Such research questions arise from the theory of embodied cognition, a theory that implies that

> changes in subjective experience, be they sensory, emotive, or intellectual, *must objectively manifest* in some way through embodied action, i.e., overt behaviour, including brain/body behaviours that have been difficult or impossible to observe

with methods traditionally available to educational researchers. (p. 318, italics inserted)

The primary way the chapter fits into the book is by raising, but by no means resolving, the question of whether embodied cognition is the only theoretical framework that can meaningfully interpret findings from neuroscience. To what degree can embodied cognition be networked with the various constructivisms in order to effectively transcend the mind/brain distinction?

The chapters in Parts VIII and IX provide two conflicting prescriptions for how to improve students' problem solving. English and Sriraman begin by showing how progress in developing student problem solving has been maddeningly slow. Research consistently shows that it does not often work to teach students problem-solving techniques, and high-stakes testing pressures effectively limit classroom emphasis on problem solving. English and Sriraman claim that research on problem solving has a poor record of accumulation and the theoretical knowledge is underdeveloped. Their solution is to use activities that require the mathematical modelling of real-life situations. Involving problems that are not tidy and sanitized of extraneous information can invoke students' meta-level reasoning and decision-making. Gerald Goldin's chapter on problem solving proposes a very different solution: Teach discrete mathematics. Discrete mathematics topics provide salient opportunities for students to develop general heuristic abilities such as "modelling the general on the particular." These are indeed clashing prescriptions — discrete mathematics problems, such as the example Goldin uses, often have contrived (or no) contexts and require no filtering of messy real-world data, but highly context-specific problems like English and Sriraman's *First Fleet* example may not foster the kind of generalizing that Goldin is championing.

This clash shows how important theory is: Because the articles do not share a common framework of what problem solving is, they provide competing prescriptions for teaching. But herein lies a significant missed opportunity for the book, for none of the commentaries points out and discusses the conflicting stances. If the whole purpose of the commentary structure of the book is to put theories into conversation with one another in order to make progress, then why are readers left with two conflicting theories simply appearing side by side with no conversation or apparent progress at all?

In Part XIV the purpose of the commentaries is clearer, taking a new look at Judith Jacobs's 1994 article about feminist pedagogy and mathematics. The article discusses some implications of an earlier theoretical framework of Belenky, Clinch, Goldberger, and Tarrule (1986, cited in Jacobs, 1994), which posits fundamental differences in the mathematical thinking of males and females. Two of these commentaries argue that since there is no appreciable gender gap in Turkey or Iceland on standardized test scores in K-12 grades (and the gap is now gone in the U.S.A. as well, according to Hyde, Lindberg, Linn, Ellis, & Williams, 2008), there is no very strong reason to believe that males and females think about mathematics in fundamentally different ways.

However, these commentaries still do not address the pronounced gender disparity in mathematics at the undergraduate and graduate levels. According to Jacobs, the Belenky et al. framework actually indicates that gender differences may not be pronounced until the relatively advanced stage of knowing in which facts are believed based on the validity of arguments (rather than, say, believed simply because the teacher said so). At this point males prefer "separate" knowing, determining an argument's validity through propositional logic and depersonalizing its statements, while females prefer "connected" knowing, determining validity by the degree to which the statements synthesize shared experiences. Might it simply be that the K-12 mathematics curriculum rarely treats students as being at the stage of knowing in which they must evaluate the validity of arguments? Surely the material on most standardized tests, in the U.S.A. at least, does not require students to be thinking at such a stage. Perhaps it is only at the undergraduate and graduate levels that such a stage is achieved, and only then does a male-oriented curriculum become alienating for many females.

Conclusion

The book addresses much more, including teachers' pedagogical actions and the politicization of mathematics education, and offers an intriguing look at covert ways of knowing. Although there is a remarkable diversity of content and perspective, the book is not an MER theory handbook meant to provide an overview of all of the field's theories. Nor is the book a detailed manual for allowing researchers to use these theories to collect and interpret MER data. The book is a conversation starter; it provides vocabularies to use, pronouncements to respond to, and examples to consider for researchers and graduate students who wish to discuss what MER theory is and should be. For these reasons, the book will be of most interest to expert, novice, and aspiring researchers who are using, crafting, refining, or reflecting upon theories for their own current and future research. Mathematics teachers at all levels may certainly find the meta-level discussion of theory interesting, particularly those who formally or informally research their own practices and are grappling with the thorny business of using theoretical frameworks to make sense of what they see.

In my opinion, the commentary structure is a partial success. Too many commentaries simply summarize the chapters or pursue tangents rather than synthesizing or reinterpreting them. This is particularly frustrating when the article has been published previously. However, the times when these commentaries do foster real discussion with and between the chapters suggests that this commentary structure holds great promise. It is most effective when the meta-level observations made in the chapters and their commentaries are grounded in specific examples, showing the reader how to incorporate these observations into their own research, thus moving the field forward rather than just laterally.

References

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