

*Theories of Mathematics Education: Seeking New Frontiers*

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Bharath Sriraman, Lyn English (Eds.) (2010)

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*Theories of Mathematics Education* is the first volume in a new Springer book series, *Advances in Mathematics Education*. The series intends to “integrate, synthesize, and extend” work in the field so that promising ideas can be improved and built upon. The seed for this inaugural book was the 2005 meeting of the International Group of the Psychology of Mathematics Education. *Theories of Mathematics Education* includes 19 essays, most with a preface and at least one commentary (a useful structure for such a wide-ranging and dense compilation of essays), collectively written by 52 contributors from 13 countries. The book pulls together the eclectic and sometimes contradictory ideas that have been advanced by theoreticians in mathematics education.

As Stephen Lerman suggests in chapter IV, mathematics education as a field of inquiry has what the linguist Basil Bernstein calls a “weak grammar,” that is, “a conceptual syntax not capable of generating unambiguous empirical descriptions.” It is not surprising, then, that the term “theory” is used throughout the book with clearly different meanings. Consider the difference between the notion of a scientific theory and a theory in literary criticism; both flavors are discussed or advocated in this book at different points, sometimes by the same author. Paul Ernest argues in chapter II that the term “theory” is often used where “philosophy” might be more appropriate. His criteria for a set of ideas to rise to the status of a theory are that it must be sufficiently specific and also testable; clearly not all contributors share this perspective. The essays fall into three categories: (1) reflections on the philosophical foundations of mathematics education as a field and how education researchers should orient themselves to their work, (2) theoretical perspectives from other disciplines that could be brought to bear on mathematics education research, and (3) descriptions of theories (or proto-theories) of the processes underlying the teaching and learning of mathematics.

Of course, this is not a perfect categorization scheme, and a number of essays could be assigned to more than one of these categories. However, chapters I, III, IV, V, and XV are primarily about the philosophical foundations of mathematics education. Chapter IV provides a framework that I found helpful in understanding some of the other chapters in the book. Stephen Lerman introduces Bernstein’s distinction between weak and strong grammars (mentioned above) as well his distinction between disciplines with vertical and horizontal knowledge structures. Vertical knowledge structures grow by integrating previous theories into new theories; horizontal knowledge structures grow by inserting new theories alongside older theories, with the possibility that they may be “incommensurate.” He suggests that science has a vertical knowledge structure

and the field of education has a horizontal knowledge structure. We will return to this framework at the end of the review.

The first chapter, written by the editors, summarizes the contents of the book and anchors it within historical work of some of the foundational thinkers in mathematics, philosophy, education, and the intersections of these disciplines. I wish I had understood this before I read the book because without this context the essay seemed to be a random agglomeration of loosely related ideas. However, after having read a significant portion of the book I went back to this chapter and it made considerably more sense to me; other readers might consider this approach as well.

In chapter III, Frank Lester attempts to address three issues: What is the role of theory in educational research? How does one's philosophical stance influence the sort of research one does? What should be the goals of mathematics education research? He argues that a "theoretical framework" enforces an ideological approach to research and that a "conceptual framework" that is "built from an array of current and possibly far-ranging sources" is a potential alternative. Mathematics education researchers, he suggests, should appropriate whatever theories seem relevant to the problem at hand, an approach known as *bricolage*. He notes the tension between theoretical and practical pursuits in educational research and proposes a model integrating them.

In chapter V, Richard Lesh and Bharath Sriraman argue that the research paradigm in mathematics education should be modeled after the "design sciences," such as architecture and engineering, which are as much about solving real problems as developing relevant theories. They suggest that a critical activity should be developing operational definitions of key concepts and note that one of the weaknesses of the research record in mathematics education is that there is little "accumulation." Whether or not researchers embrace the idea of mathematics education as a design science, their call to develop operational definitions and to work towards accumulation is a call to develop theories of mathematics education with a stronger grammar and vertical knowledge structures.

In chapter XV, Angelika Bikner-Ahsbals and Susanne Prediger address the plurality of theories in mathematics education and the criticism that mathematics education research lacks focus by describing ways to link different theories together by "networking of theories." They begin by noting that people use the word theory in different ways and review some efforts by scholars to define the term more precisely and describe the various roles that theories play. Like several authors in this book, they trace this multiplicity of theories to the complexity of the systems being studied. They then suggest various approaches to knitting theories together. In chapter XVI, Helga Jungwirth and Uwe Gellert present two detailed examples of the application of such networking. Gellert also provides a much-needed critique of the *bricolage* approach.

Five chapters present theoretical perspectives from other disciplines. In chapter VII, Luis Moreno-Armella and Bharath Sriraman advocate attending to the semiotic dimension of mathematics. In chapter X, Stephen Campbell discusses integrating cognitive neuroscience and psychophysiology with educa-

tional research. In chapter XIV, Judith Jacobs brings a feminist perspective into the mix. In chapter XVII, Andy Hurford suggests ideas from complexity theory that could be brought to bear. In chapter XVIII, Nathalie Sinclair suggests that the study of gesture, intuition, and aesthetics can inform our understanding of tacit (as opposed to explicit) mathematical knowledge.

The remaining chapters provide descriptions and applications of theories (or proto-theories) related to teaching and learning mathematics. In chapter II, Paul Ernest describes different variants of constructivism. In chapter VI, John Pegg and David Tall review theories of mathematical concept formation. In chapter VIII, Gerald Goldin suggests that discrete mathematics is a useful mathematical discipline for teaching problem solving heuristics, while in the commentary for this chapter and the preface for the next, Jinfa Cai points out that past theories about the development of problem solving are problematic and that new research is very much needed. In chapter IX, Lyn English and Bharath Sriraman develop this theme at length, and suggest directions for future research on problem solving. In chapter XI, Guershon Harel describes his *DNR* framework for mathematics instruction. In chapter XIII, Günter Törner, Katrin Rolka, Bettina Rösken and Bharath Sriraman discuss an application of Alan Schoenfeld's theory of *Teaching-in-Context*. Finally, in chapter XIX Bharath Sriraman, Matt Roscoe, and Lyn English discuss the politics of mathematics education. Given the wide-ranging nature of these chapters it is not possible for me to do them justice in this short space; however, the interested reader should now have a better idea of where to begin.

Several themes recur; two in particular caught my attention. The first is the issue of a plurality of theories *versus* a “grand theory.” Much is made of the complexity of educational settings. However, there is also tremendous complexity in biological systems, for example, and yet there are many useful theoretical structures in that discipline. It is instructive to briefly compare the two areas of inquiry. As with mathematics education, there is a multiplicity of theories in biology both within and across different levels of analysis; for example, there are theories about processes within cells, theories about the structure and function of nervous systems in organisms, theories about habitats for individual species, and theories about ecosystems. Note, however, that there is just one theory about the structure and function of the nervous system of the frog. The multiplicity of theories in biology are meant to explain different phenomena; there are not usually multiple theories for the same phenomenon, and when there are, there is an effort to sort out which one is most consistent with both the available empirical evidence as well as neighboring or overlapping theories. Furthermore, the layered nature of biological theories allows that they all be subsumed within the grand theory of biology, namely, the theory of evolution. While we are nowhere near such an elaborately structured set of theories in mathematics education, the complexity of educational systems does not preclude the possibility that a well-articulated set of linked and nested theoretical structures could be developed over time. I would argue, furthermore, that this should be a long-term goal of researchers in the field.

The second (clearly related) theme is the question of whether research in

mathematics education is or should be “scientific” or at least more scientific. I would have liked to see this issue addressed more directly, though the contributions of several authors shed light on what this would mean and how it might be accomplished. A clear implication of Lester’s interpretation of Bernstein’s theory is echoed by Lesh and Sriraman: for mathematics education research to become more scientific, mathematics educators must work to develop more tightly defined concepts (i.e. a stronger grammar for the field) that are organized within vertical knowledge structure (i.e. that lead to accumulation). The discussion of theories about and research into problem solving in chapters VIII and IX is one of the few examples in the book of a research area that reflects this scientific process. Early theories about how problem solving works and should be taught that sprang from the heads of philosophically minded people were initially embraced by practitioners and researchers. However, the empirical record has shown that those theories are problematic, and researchers are now calling for revisions as well as new research. The insights gained through this accumulating process and the forward planning suggested by the authors and commentators of these chapters should serve as a model for the work of mathematics educators.

As Lesh and Sriraman point out, mathematics education as a field of inquiry is still in its infancy. *Theories of Mathematics Education* represents a much needed effort to bring some coherence to the theoretical foundations of this young field, although it is safe to say that there is still a tremendous amount of work to be done. The authors have laid the groundwork for a natural next step, namely, a systematic survey of the big questions in mathematics education that need to be addressed. This survey should include a discussion of research methods that might be appropriately used to investigate them and weaknesses in both the relevant empirical record and extant theories, many of which are still very immature and should necessarily be refined as time passes. The ultimate test of the value of the ideas in this book is whether they or their progeny help solve the problems that teachers, administrators, and policymakers face as they work to improve mathematics teaching and learning.