Theories of Mathematics Education edited by Bharath Sriraman and Lyn English: Common Ground for Scholars and Scholars in the Making[1]

Sriraman, B. & L. English (Eds.) (2010). *Theories of mathematics education: Seeking new frontiers (Advances in Mathematics Education)*. Berlin/Heidelberg: Springer Science. ISBN: 978-3-642-00741-5. 668 pages, $129.

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Theories of mathematics education: Seeking new frontiers is the first book in a new Springer series, *Advances in Mathematics Education*. To some degree the book is based on a collection of previously published papers from special issues of *ZDM – The International Journal on Mathematics Education* (previously known as *Zentralblatt für Didaktik der Mathematik*). These papers, dealing with the role and use of theories in and about mathematics education, originally stem from various conferences and meetings such as for example PME and CERME. For this reason some of the papers in the book are already well known, a few of which may even be considered to be 'modern classics' within theories of mathematics education, such as, for example, Frank Lester's *On the theoretical, conceptual, and philosophical foundations for research in mathematics education.* What is new – and non-traditional – in the book, however, is its form of presentation and format, the main articles being accompanied by preludes and commentaries by established researchers as well as newcomers.

My review of the book will be divided into five parts. In the first I give a brief overview of the book and its contributors. In the second part I try to articulate my own experience in reading the book, including particularly my view on the book's special format with commentaries and preludes. In

[1] This review has also appeared in a Danish and slightly different version in the journal of *Nordic Studies in Mathematics Education*, 15(3).

the third part of the review, I address the possible benefits that doctoral students might have from reading a book about theories of mathematics education like this. And in the fourth part I provide a similar attempt to line up some of the book's topics that ought to be of interest and concern to mathematics education scholars in general. At the very end I shall make some summarizing and concluding remarks.

The contents of the book

The book consists of 19 parts all in all, each addressing different aspects of the role of theories and theory building in mathematics education. To give an idea of the scope of the book and its contributors, I briefly mention the contents of each of its parts.

Part I is a survey by the book's editors of theories and philosophies in mathematics education with a prelude by Jeremy Kilpatrick. In Part II Paul Ernest provides a short introduction to constructivism and its role in mathematics education. Commentary on this article is offered by Simon Goodchild and Paul Ernest himself. Part III is the previously mentioned article by Frank Lester, which is commented upon by Guershon Harel. In Part IV Stephen Lerman poses the question of whether the plurality of theories in mathematics education is a problem or not. This part has a prelude by Norma Presmeg and a commentary by Eva Jablonka and Christer Bergsten. Part V also contains a paper that may already be known to some readers, namely Richard Lesh and Bharath Sriraman's "Re-conceptualizing mathematics as a design science." This article is accompanied by no less than three commentaries by, Miriam Amit, David N. Boote, and Claus Michelsen, respectively. In Part VI John Pegg and David Tall discuss the fundamental cycles of concept construction which seem to be underlying several theoretical frameworks. The prelude is by Stephen J. Hegedus and the paper is thoroughly commented upon by Bettina Dahl.

Part VII consists of an article about symbols, mediation, and semiotics in mathematics education by Luis Moreno-Armella and Bharath Sriraman with prelude by Stephen J. Hegedus and commentary by Gerald A. Goldin. Parts VIII and IX concern problem solving in mathematics; Part VIII is by Gerald A. Goldin and discusses "Problem solving heuristics, affect, and discrete mathematics" with a commentary by Jinfa Cai; in Part IX the editors take stock of the field of problem solving and its future. This part is commented upon by Peter Grootenboer and Alan Zollman. With the contribution “Embodied minds and dancing brains” in Part X Stephen R. Campell discusses the possibilities of using neuroscience in mathematics education. This part has a prelude by Layne Kalbfleisch and a commentary by Scott Makeig. In Part XI Guershon Harel presents his ’DNR-based instruction’ (DNR = Duality, Necessity, and Repeated Reasoning) and discusses the role of mathematics in research frameworks for mathematics education. This part is introduced by Moreno-Armella and commented on by Bharath Sriraman, Hillary VanSpronsen, and Nick Haverhals. Part XII contains a contribution by Stephen Hedegus (no prelude and no commentary) in which he brings forth qualitative research in mathematics education and the need for ’scientificity’. Part XIII offers the article “Understanding a teacher’s actions in the classroom by applying Schoenfeld’s theory ‘teaching in context’: reflecting on goals and beliefs” by Günter Törner, Bettina Rösken, Katrin Rolka, and Bharath Sriraman with a prelude by Goldin and a commentary by Dina Tirosh and Pessia Tsamir. Part XIV is on “Feminist pedagogy and mathematics” by Judith E. Jacobs. This quite interesting piece is introduced by Gabriele Kaiser and commented on by Gilah Leder, Safure Bulut, Bekir S. Gür, Bharath Sriraman, and Gudbjörg Pálsdóttir.

Parts XV and XVI are about networking of theories, one of the book’s newer and more interesting themes. In Part XV Angelika Bikner-Ahsbahs and Susanne Prediger offer strategies on how to combine different theories in mathematics education. Their article has a prelude by Tommy Dreyfus and is put into a broader perspective by Ferdinando Arzarello’s commentary. Part XVI provides two concrete examples of networking of theories, first with Helga Jungwirth and next with Uwe Gellert, whose contribution is commented upon by Tine Wedege. Part XVII offers an article by Andy Hurford about “Complexity theories and theories of learning: literature reviews and syntheses” with a prelude by Richard Lesh about “The importance of complex systems in K-12 mathematics education” (this part contains no commentaries). In Part XVIII Natalie Sinclair discusses: “Knowing more than we can tell.” This part has an introduction by Bharath Sriraman and a commentary by David Pimm. In Part XIX we finally touch upon the more political aspects of mathematics education with a contribution by the editors in corporation with Matt Roscoe and with a commentary by Keiko Yasukawa.

The book’s form of presentation

As can be deduced from the above, the book contains various inputs from a long line of prominent people within the area of mathematics education, but as mentioned, one of the more special features of the book is its format with preludes and commentaries connected with the 20 main contributions of the books’ 19 parts (two articles in Part XVI) and the variety of young and old contributors. The interesting question, of course, is to what extent the book’s non-traditional form is a success or not.

From an overall perspective, the format as implemented in the book is somewhat uneven. As already suggested, there seems to be no consensus regarding the number of commentaries in each part, or whether every part should contain a commentary or prelude at all. However, this is a minor aspect as opposed to what in my mind constitutes the more frustrating experience one has as a reader: the difference in nature between commentaries (and preludes also). Some commentaries only provide a small summary of the article just read, which in itself seems redundant, and this in particular if the article was introduced in the prelude in a similar fashion. Sometimes this problem is ‘solved’ by providing several commentaries to an article, which of course does not reduce the length of the already quite long book. Of commentaries in the very opposite end of the spectra are those where, perhaps after summarizing the article briefly, a researcher takes the opportunity to introduce his or her own research program, thus causing the commentary almost to exceed the original paper in both length and scope. Or even worse, there are one or two preludes where researchers manage to present their own view on things without as much as mentioning by a single word the article that they are to introduce.

However, having said this, the book contains several examples of places where its structure functions really well. In particular this is so where researchers are capable of placing an article within a larger context or truly draw perspectives over its contents (as opposed to picking up and following some perhaps/perhaps not related tangent). One example is when David N. Boote in Part V asks what the consequences would be for the field of mathematics education if indeed it were re-conceptualized as a design science, as suggested by Richard Lesh and Bharath Sriraman. Or when Ferdinando Arzarello in Part XV places the idea of networking of theories into the greater context of what he, referring to Luis Radford, calls a semiosphere. Another good example is Ewa Jablonka and Christer Bergsten's commentary to Part IV in which they decide to 'test' out Stephen Lerman's results by looking into recent issues of *Educational Studies in Mathematics* and *Journal for Research in Mathematics Education*. Or, as a last example, when Guershon Harel in his commentary to Part III as a consequence of Frank Lester's article draws the conclusion that research frameworks in mathematics education should include or build on some form of mathematical context (a matter I will address later).

Just as some of the commentaries function better than others, some of the book's parts function better and rise above others. The best example of this is the two parts (XV and XVI) on networking of theories, which neatly complement one another. Angelika Bikner-Ahsbahs and Susanne Prediger describe in general terms a set of overall strategies for combining theories in mathematics education studies. The illustration hereof comes through the papers of Helga Jungwirth and Uwe Gellert, who each show the possible gains of actually applying such strategies in practice. Uwe Gellert's combination of semiotics and pedagogical structuralism, for example, leads to a richer analysis of a classroom situation than the two theories can provide on their own and furthermore results in the posing of new research questions and development of theory. In this way, these two parts also relate directly to Part IV as a positive outcome of the plurality of mathematics education theories. Another example of this is Guershon Harel's paper in Part XI which takes Frank Lester's article from Part III as its point of origin and builds on this. It is exactly when the parts of the book in one way or another succeed in relating and drawing connections to each other, and do not just appear as isolated islands, that the book is at its best.

Where doctoral students might benefit from reading this book

One question that you are likely to encounter as a doctoral student within mathematics education is *why you need a theory*, or several theories, in the first place in order to answer your research questions. That this is so may be inferred from the constant presence of themes on the role of theories and theoretical frameworks in summer schools, workshops and seminars for doctoral students such as the European YESS and YERME and the Nordic NoGSME.[2] But even accepting the need of a theory, doctoral students will be left to ask *which theory or theories should they apply* in their studies, and having done that *how are these then actually applied in practice*.

Obviously there are no unique answers to the above questions, which of course also have to do with the fact that theories may serve different purposes. According to Mogens Niss (2007, pp. 1308-1309), theory may serve six different purposes: (1) it may provide *explanation* of some observed phenomenon; (2) it may provide *predictions* of certain phenomena; (3) it may provide *guidance for action and behavior*, for example, in specific implementations in order to achieve certain goals; (4) it may provide a *safeguard against unscientific approaches*, enabling us to avoid "haphazard and inconsistent choices with regard to terminology, research methodology, and interpretation of results"; (5) it may provide *protection against attacks from the outside*, that is, attacks from researchers in other scientific disciplines questioning the foundations and results of our research; but most importantly, (6) it may provide *a structured set of lenses* through which phenomena may be "approached, observed, studied, analyzed, or interpreted". Concerning this latter purpose, Mogens Niss continues:

> This takes place by selection of the elements to be considered important in the context, by focusing on certain features, issues, or problems; by adopting and utilising particular perspectives, and providing a *methodology* for answering questions concerning the domain(s) considered. (Niss, 2007, p. 1309)

The close relationship between theory and method is also addressed by Edward A. Silver and Patricio G. Herbst (2007). In addition to some of the points made by Niss, they also mention the use of theory as "a tool that helps give closure to the corpus of data to study a problem, complementing the sources identified by the problem" (Silver & Herbst, 2007, p. 50).

As a doctoral student, it may happen that you work within a certain tradition, for instance if the research of your supervisor lies within an already well-established tradition in mathematics

[2] YERME = Young European Researchers in Mathematics Education; YESS = YERME Summer School; NoGSME = Nordic Graduate School in Mathematics Education.

education, as for example, the French traditions of TDS or ATD or the Dutch RME.[3] In such a case the student's research will very often be what Abraham Arcavi (2000) describes as *theory-driven*, that is, research that in one way or another tries to underpin, test, or expand an already existing theory. Typically this happens by trying to answer questions that are posed within the theory itself – thus, it is the theory that gives rise to the research question. In addition to theory-driven research, Abraham Arcavi (2000, p. 145) mentions *problem-driven* research, claiming that a distinction between the two "help[s] to pinpoint the main initial impetus" of what researchers in mathematics education do (though most may use a "dialectic integration of the two"). Also Alan H. Schoenfeld (1992) points to these two types of research and further mentions a third, namely *method-driven* research where focus is on the examination of a certain method, for example, by testing this in a new setting, situation, or within a different problem field, etc. Besides these three types, Uffe Thomas Jankvist (2009) mentions a possible fourth type which is *data-driven* research, research that would take its departure point in already accumulated data, for example, by trying to explain phenomena in studies such as PISA or TIMSS.[4]

In particular for students who are involved in problem-driven research, the task of finding and choosing a theory is very critical, and this maybe even more so if the student defines his or her own research and formulate his or her own research questions, for example, based on one's own teaching practice or experiences from a previous mathematics graduate study – something which is at least the case for many Nordic PhD-students. In such a case, where the research is more problem-oriented, it is often far from a priori given which theory (or theories) should or could play a part in the individual student's PhD research. It is my belief that a book such as that by Bharath Sriraman and Lyn English can assist in clarifying the role of theories in mathematics education research for a doctoral student by providing (i) information and enlightenment, (ii) examples and illustrations of different theories and their use, and last but not least (iii) inspiration on how to include and use theoretical elements in a sensible manner in the answering of one's research questions, clarifying in that way which of Niss' six purposes the use of theory is to play in one's

[3] TDS = Theory of Didactical Situations; ATD = Anthropological Theory of Didactics; RME = Realistic Mathematics Education.

[4] PISA = Programme for International Student Assessment; TIMSS = Trends in International Mathematics and Science Study.

own research. I shall illustrate each of these three possible types of benefits (i, ii, iii) with examples taken from the book.

As an example of a particularly enlightening article about the role of theories in mathematics education research, I have chosen Frank Lester's paper in Part III, partly because I found this highly informative and clarifying when I was myself a Ph.D. student. Frank Lester addresses the role of theory in mathematics education research through a discussion of the nature of research frameworks. Frank Lester (p. 69) uses the metaphor of a *scaffold* for a framework: "A scaffold encloses the building and enables workers to reach otherwise inaccessible portions of it. Thus, a research framework is a basic structure of the ideas (i.e., abstractions and relationships) that serve as the basis for a phenomenon that is to be investigated".[5] Such a research framework "helps us develop deep understanding by providing a structure for designing research studies, interpreting data resulting from those studies, and drawing conclusions." Inspired by the anthropologist Magaret Eisenhart, Frank Lester distinguishes between three kinds of frameworks: *theoretical*, *practical*, and *conceptual*. Within theoretical frameworks the researcher uses accepted conventions of argumentation and experimentation associated with the theory, and research questions are rephrased in terms of this and to some extent determined by it. The goal of the research and the data it gathers is to support, extend, or modify the theory in question – very much in line with the observations of both Alan Schoenfeld and Abraham Arcavi. Although approaching research in mathematics education through theoretical frameworks has the advantages of "facilitating communication, encouraging systematic research programs, and demonstrating progress", it also has the serious disadvantages of forcing researchers to explain their results by 'decree' rather than evidence, making data 'travel' to serve the theory, not setting a discourse helpful in everyday practice, and not offering enough opportunities for validation by triangulation (p. 71). A practical framework is based on accumulated practice knowledge and 'what works'. One advantage of practical frameworks over theoretical frameworks is that the problems addressed are those of the people directly involved. However, a drawback is that the

[5] As made clear to me by Michael Fried, when he read this piece, the more famous use of the 'scaffold' metaphor of course is that of Brunner to describe Vygotskian mediated learning, where the point of the metaphor was that as a scaffold is a temporary structure so also the added support given by a teacher will eventually be removed. A framework, however, is not necessarily temporary, and generally is not – something that Frank Lester does not seem to take into consideration when drawing his parallel to a scaffold.

practical frameworks suffer from being, at best, only locally generalizable, for example, because of narrow insider perspectives. Conceptual frameworks focus more on justification than on explanation and base themselves on both previous research and theory. However, instead of relying on only one overarching theory, as with theoretical frameworks, they build on a variety of sources and can be “based on different theories and various aspects of practitioner knowledge, depending on what the researcher can argue will be relevant and important to address about a research problem” (p. 73). With reference to Kueno Gravemeijer and Claude Levi-Strauss, Paul Cobb (2007, p. 29) refers to such a scenario as a *bricolage*, a term relating to the French *bricoleur*, a handyman skilful of using whatever tools available in a given situation. Thus, in conceptual frameworks, as well as in a *bricolage*, it is the problems which drive the research and helps identify the theoretical constructs used to build the scaffold. One problem of taking such a *bricolage* or conceptual framework approach is, as also indicated by Paul Cobb (2007, p. 31), that of co-existence and conflict. That is to say the applied theoretical constructs must be able to co-exist in such a manner that they do not conflict with or contradict one another. These are aspects that are discussed often in parts of the book on networking of theories and which therefore could be very useful to read if one had to decide on the use of a certain conceptual framework.

Besides the examples from Part XVI of networking of theories, which I mentioned above, the book also provides a couple of other examples of different frameworks in use. One such is Guershon Harel’s *DNR-based instruction* (DNR = Duality, Necessity, and Repeated Reasoning) in Part XI, which as suggested by its name is a framework for the teaching of mathematics – and furthermore one which is based on empirical research. In addition to explaining his frameworks’ premises, concepts, and principles for instruction Guershon Harel also provides us with an elaborate example in the form of a classroom situation which he interprets and analyzes using his three principles (D, N, and R). Part XIII also gives us a different example of a theory in use in the form of Schoenfeld’s *teaching-in-context*, which concerns the decisions in a teaching situation that may change the teacher’s planned discourse as well as how the decisions that are being made depend on the individual teacher’s knowledge, goals, and beliefs (KGB). Günther Törner et al. describe such a case in a German 8^{th} grade class. As pointed out by Dina Tirosh and Pessia Tsamir in their commentary, the article is an example of how a theory may be applied within a different setting than the one for which it was originally developed; the commentary also stresses that the article shows how two different theories can come to support each other, namely, that of Alan Schoenfeld and Lee S. Shulman's *theory of teachers' knowledge*.

Every one of the above-mentioned chapters can of course be a source of inspiration for doctoral students and provide them with ideas for their own research. A different form of inspiration may, however, be available from the two parts on problem solving (Parts VIII and IX). These provide fundamentally different suggestions on how to best develop students' problem solving abilities. Where Lyn English and Bharath Sriraman point to activities involving mathematical modeling of real-life and to some degree complex situations and events, Gerald A. Goldin is of the opinion that the solution lies in having students study discrete mathematics. According to Gerald A. Goldin studying the discrete structures provides the opportunity for one to develop his or her own heuristic abilities and by that become able to model the general on a basis of the concrete. On the one hand this shows that there may be different approaches to the same didactical problem. On the other hand, it also clearly shows that an underlying disagreement of what a mathematics educational concept such as problem solving is to cover – and theories about this – may lead to very different conclusions to the same problem – something that may inspire students to go in other directions than just the traditional and already established ones.

In general, the book will expand doctoral students' mathematics educational horizon by exposing them to many lesser-known aspects of the field and raising quite a few questions along the way, which newcomers to the field might very well be unfamiliar with – in other words, the book may serve the purpose of providing some mathematics education *Allgemeinbildung*. Among the questions the book brings out are the ones centered on meta-perspectives, questions addressing elements such as mathematics education as an academic discipline, its relations to other academic fields, and a status over its results within various sub disciplines. Although such questions may be more directed towards the more established and experienced researchers of the field, I do believe that PhD-students should at least be cognizant of their existence.

Benefits for scholars in mathematics education in general

Given that more established researchers ought to have a clearer perception of the role of theories in mathematics education research they are more likely to benefit from the parts of the book that address theories *about* mathematics education than those that address theories *in* mathematics education. And the interested reader will find several such theories about mathematics education. I shall not go into a long and detailed discussion of all these here however. Instead I shall point to a handful of questions and problems, which in my reading of the book, I found particularly salient and which I believe that scholars in mathematics education research should take note of and consider seriously.

Among the more predominant themes in the book are those concerning the connection between theory and practice as well as the feedback mechanisms between these, for example, how to accumulate and process the knowledge and experiences obtained through research, how to apply these in the development of theories, and how to put theories into practice through new experiments. Another predominant theme concerns the recurring and often debated question of whether mathematics education should strive for a grand unifying theory or not, if such a theory is at all obtainable, and if so, what our benefits from it could actually be. A third example concerns the discussion of 'what works', which to some extent is brought on by the political steps regarding education especially in the USA, which have had massive consequences for American educational research. Several of the book's contributors point out the dangers of such an approach and make it very clear that knowing only *what* works is far from enough; we also need to know *why* it works, *how* it works, *when* it works, and when it does *not*. Besides these more well-known meta-themes of mathematics education, the book also offers others. I shall outline two of these that I found to be interesting as well as important for the community and its researchers to consider.

One example of such a meta-theme is the one which Guershon Harel raises, first in his commentary to Part III and then again in his article in Part XI. Guershon Harel (p. 343) states that, "…attention to mathematical content is peripheral in many current frameworks and studies in mathematics education." He then underpins his claim by making the following observation:

> ... many current studies, rigorous and important in their own right as they may be, are adscititious to mathematics and the special nature and the learning and teaching of mathematics. Often, upon reading a report on such a study, one is left with the impression that the report would remain intact if each mention of 'mathematics' in it is replaced by a corresponding mention of a different academic subject such as history, biology, or physics. There is a risk that, if this continues, MER [mathematics education research] will likely lose its identity. (p. 343)

According to Guershon Harel this problem is best solved by enhancing the quality of the mathematical components in graduate and doctoral programs of mathematics education (p. 87), or by requiring that students who enroll in the mathematics education programs come with a strong mathematical background. One must ask though, to what extent the more established researchers in mathematics education are willing to take these steps and whether such measures taken to strengthen the identity of MER would then have a lopsidedness about it. One of the things this approach might threaten is the plurality which mathematics education offers. It is this kind of plurality Ewa Jablonka and Christer Bergsten (p. 116) argue for when they say: "We want to suggest here to continue to pay serious attention to developments outside the field of mathematics education in order to advance theory." One of the ways (out of several) of doing this is exactly by recruiting competent students with a different background than just a mathematical one alone. However, I do believe that Guershon Harel makes a very strong point: where would we be if, for example, mathematics educators could not themselves teach courses in mathematics and if they lacked understanding of the mathematical structures the students they are investigating are supposed to be learning?

A different meta-theme certain elements of the book touch upon is that concerning whether mathematics education is or is not an academic discipline. Jeremy Kilpatrick (2009) discusses the theme in the proceedings from the Rome conference of the 100th anniversary of ICMI (Menghini et al., 2009). The theme also pops up indirectly as when David N. Boote asks what consequences would follow from adopting the steps proposed by Richard Lesh and Bharath Sriraman in Part V and re-conceptualizing mathematics education as a design science:

> ... within neo-liberal countries that do not value craft, the enterprise of educational research is hampered by the perception that educational research is a soft science [...]. In these countries the modest rise of prestige of teacher educators in higher education has been tied to our publication of basic research [...]. Typically, design fields – engineering, architecture, urban planning – have lower prestige than the basic sciences, or even the social science and humanities. How will our position in higher education be affected if we shift to design science instead of basic science? Our ability to produce what seem like generalizable knowledge claims gives us a modicum of credibility and prestige within higher education; it is not difficult to imagine that a shift towards design science will diminish our place in higher education. (p. 163)

A related question is to what extent the more established researchers within mathematics education would actually be willing to give up the possible status of their publications as being 'basic research', even if they were in fact convinced that the field of mathematics education and its research results could benefit greatly from such a shift in discourse towards a design science. In my opinion this makes for an interesting meta-theme (or meta-meta-theme), since it offers an opportunity to reflect about the state of our field as well as the goals that we set for our research and work.

Final remarks

Allow me to briefly summarize some of the points made above as well as to give a few final remarks on the book and this review both.

As mentioned, the way the book implements its decided structure of preludes and commentaries as well as the degree to which these actually introduce articles, comment on articles and the meta-themes connected with them, or just follow some tangent is very non-uniform. In my opinion a greater synergy would have been preferable: that could have illuminated how the different parts of the book are connected or stand in opposition to one another – an exercise almost completely left to the reader. Probably such a task would have demanded that the editors write the commentaries themselves since it would require quite a profound overview of all the book's 19 parts. Doing so would of course have had as a consequence that the book would stand to lose its many voices, and I assume that this is one of the main reasons for rejecting this option. From the editors' side, I think, that precisely the many – and various – voices is a main point of the book itself. As should be obvious by now, Bharath Sriraman and Lyn English's book is not a handbook of theories of mathematics education, nor is it exactly to be considered a traditional source book. Far more, it is a book of conversation and debate, which I mean in the sense that it is a book that prepares the ground for more conversation and debate about the role and state of theories in mathematics education. And with this purpose in mind I believe that the book is indeed a success.

As discussed earlier, the book also contains various opportunities for newcomers within the field to see how theories may be put into play and by that get inspiration for one's own research. My own one-time doctoral advisor, Mogens Niss at IMFUFA, Roskilde University 'brought up' his graduate students to believe as a doctoral student in mathematics education they have to "think for themselves", for no cookbook recipes are given within this field of research. Mogens Niss himself recently expressed it in this way:

> ... there is no established royal road to guaranteed quality or relevance in mathematics education research. In that sense every PhD project is unique, which puts high demands on graduate students' independence and originality. Every PhD student must "think from scratch", even if (s)he is supervised by one of the most experienced, renowned and respected supervisors. (Niss, 2010, p. 22)

This means that doctoral students have no book of answers at their disposal which will tell them how to choose theory (or theories) for their research work – nor is there one telling how to argue for the relevance of a theory once one has been chosen. But this is where the book edited by Bharath Sriraman and Lyn English in my mind has something to offer doctoral students, for while it demonstrates that neither choice nor arguments for theories can be unequivocal, it provides nevertheless several examples of how to approach research in the manifold realm of theoretical constructions that mathematics education has become.

If we, although perhaps with a twinkle in our eye, imagine that we could take "to think for yourself" as a 'ground principle' for mathematics education research, then perhaps a second one might be that of another IMFUFA researcher, physics educator, Jens Højgaard Jensen. The motto for his educational research, by his own account, is “didactics should be honest talk”.[6] By this, Højgaard Jensen is of course also referring to the political agenda, which the book addresses frequently too, of attempting to make the humanities and social sciences in general and the educational sciences in particular adapt the same research paradigms as the more hardcore natural sciences. Barath Sriraman and Lyn English’s book occasionally offers elements of what I will consider to be straightforward ‘honest talk’ – something that in my mind offers great relief as a reader. One example is when David N. Boote in his commentary to mathematics education as a design science comes straight out and says: “Poorly educated mathematics teachers are, arguably, a far greater problem for student learning than the failings of mathematics education research.” (p. 163) Another example of such ’honest talk’ is when Tommy Dreyfus comments on the possible reasons for the plurality of theories in mathematics education and points to the following as being a possibility one cannot ignore: “Indeed, it appears that many researchers tend to prefer developing their own frameworks rather than read, learn, understand, adopt, adapt, and apply existing ones that were developed by others.” (p. 479) As a final example of ’honest talk’, there are of course several others throughout the book; I mention that of Bettina Dahl, when she says: ”we must not only discuss the theories in relation to each other ... but equally important is it to collect more empirical data that might cast light on these discussions” (p. 206) In my opinion Bettina Dahl hits the nail on the head here, because it is exactly in the cases where the book’s contributors to some extent manage to provide empirical evidence for their claims or for their use of a certain theory that you are convinced the most as a reader and attracted the most as a researcher.

So to round off in a more statement-kind-of-way: this book with its 668 pages is a ‘brick’ in the physical sense of the word, but, I believe, it may also become a ‘brick’ in a different sense as well, namely, as an important building brick in the ongoing construction of a theoretical foundation for the field of mathematics education.

[6] I am aware that these two ‘principles’ are very general as well as ideal and would apply to a long line of academic research, not only mathematics and science education.

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