Math 422: Abstract Algebra II
Homework 7, Due Friday, March 21

- Chapter 14: # 28, 29, 30, 32, 39.

Let k be a field and let $k[x_1, \ldots, x_n]$ denote the polynomial ring in n variables. If I is an ideal of $k[x_1, \ldots, x_n]$, let

$$V(I) = \{(a_1, \ldots, a_n) \in k^n | f(a_1, \ldots, a_n) = 0 \text{ for all } f \in I\},$$

and, for $S \subset k^n$ let

$$\mathcal{I}(S) = \{f \in k[x_1, \ldots, x_n] | f(a) = 0 \text{ for all } a \in S\}.$$

1. Prove that $\mathcal{I}(S)$ is an ideal of $k[x_1, \ldots, x_n]$.
2. Prove that, if $I, J \subset k[x_1, \ldots, x_n]$ are ideals with $I \subset J$, then $V(J) \subset V(I)$.
3. Let $g_1, \ldots, g_m$ be elements of $k[x_1, \ldots, x_n]$, and let

   $$W = \{a = (a_1, \ldots, a_n) \in k^n | g_1(a) = \cdots = g_m(a) = 0\}.$$

   Prove that $V(< g_1, \ldots, g_m >) = W$ and $V(\mathcal{I}(W)) = W$.