Math 422: Abstract Algebra II
Spring 2008

Meeting Information: Monday, Tuesday, Wednesday, Friday, 1:10-2:00 in Math 108.
Instructor: Adam Nyman
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Email: nymana@mso.umt.edu
Office hours: (Tentative) Mondays 2:10-3, Tuesdays 2:10-3, Fridays 12-1 and by appointment.

Textbook: *Contemporary Abstract Algebra, Sixth Edition* by Joseph A. Gallian

Class Web Page: http://www.math.umt.edu/Nyman/math422.mht. All homework and handouts will be available only on this website.

Prerequisites: MATH 421.

Overview: We will continue the study of groups, rings, and fields begun in MATH 421. In particular, we will first develop enough group theory to appreciate the overall strategy of classifying finite groups. We will then study the classification of finite abelian groups. That is, we will write a "short" list of finite abelian groups with the property that *any* finite abelian group is isomorphic to one of the groups on our list.

Galois Theory is the study of the symmetries of the roots of polynomials. It can be used to describe exactly when the roots of a polynomial are given by a formula involving addition, subtraction, multiplication, division and root extraction of its coefficients. For example, if $f(x)$ is a quadratic polynomial, such a formula for the roots of $f(x)$ exists: it is the quadratic formula. One can show that formulas exist for cubic and quartic polynomials as well. One of the amazing consequences of Galois Theory is that there exists no such formula for polynomials of degree 5 or greater. In the second part of the course, we will study enough ring and field theory so that we can learn the rudiments of Galois Theory.

Exams: There will be three fifty minute exams: the first is on Tuesday, February 19, and will cover the material on groups (see the outline on the reverse), and the second is on Tuesday, April 1 and will cover the material on rings and vector spaces. The third (fifty minute) exam is during the final exam time slot on Monday, May 5, 3:20-4:20 and will cover the material on fields and Galois Theory.

Homework: There will be weekly homework due, for the most part, Wednesdays at the beginning of class. While you are encouraged to discuss the homework with classmates, you must write up solutions without assistance. Tuesdays will generally be spent discussing homework.

Late Homework Policy: Each of you will have one opportunity to hand in late homework (no later than two class meetings after it was due). After you have used up your opportunity, no late homework will be accepted. Since there will be no exceptions, and as a courtesy to your grader, please try to use these only for emergencies.

Grading: Your grade will be based on homework and three exams as follows:

- homework: 31 %
- exams: 23 % each

Other Remarks:

- Students with disabilities are welcome to discuss accommodations with me.
- All students need to be familiar with the Student Conduct Code. You can find it in the "A to Z Index" on the UM home page.
- All students must practice academic honesty. Academic misconduct is subject to an academic penalty by the course instructor and/or a disciplinary sanction by the University.

Outline: We will cover the following topics, most of which can be found in the text.

- **Groups**
  - Review of Groups
  - Chapter 8: External Direct Products of Groups
  - Chapter 9: Normal Subgroups and Factor Groups
    - Normal Subgroups
    - Factor Groups
    - Applications of Factor Groups
  - Chapter 10: Group Homomorphisms

    Remark: We will go over the material in Chapters 8-10 rather quickly, as it should be a review.
  - The Classification of Finite Groups
  - Chapter 11: Fundamental Theorem of Finite Abelian Groups
- **Rings**
  - Chapter 14: Ideals and Factor Rings
  - Theorem 17.5: Irreducibility and Maximal Ideals
  - Chapter 15: Ring Homomorphisms
    - Definitions and Examples
    - Properties of Ring Homomorphisms
- **Fields**
  - Chapter 19: Vector Spaces
  - Chapter 20: Extension Fields
  - Chapter 21: Algebraic Extensions
  - Chapter 32: An Introduction to Galois Theory